AN INVESTIGATION OF A TWIN-TRIODE BALANCED MODULATOR CIRCUIT

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PREFACE

This thesis reports work preformed in an investigation of the potentialities of a twin-triode modulator circuit. The investigation included linear and nonlinear theoretical analysis, and experimental verification of predicted characteristics of the circuit.

The prediction of the performance of a modulator, in anything but the most rudimentary circuits, is a difficult problem, since it must inherently be based on nonlinear circuit analysis techniques. For many years, the approach to such analyses has been through the "power-series" representation of a nonlinear function. In any but the most simple circuits, this approach quickly results in high-order algebraic polynomials in several variables which require simultaneous solution.

This investigation introduces a possible alternate approach by the "perturbation" of selected circuit parameters in such a manner as to retain physical interpretations of the various quantities. As the need for simplifying assumptions arises, they can be based on an evaluation of the physical importance of the individual quantities.

In the process of this investigation, it was discovered that linear circuit analysis, which can tell nothing of the capabilities of a circuit as a modulator, may reveal other fundamental properties of a circuit which are of importance. This is particularly true in the analysis of the inherent "carrier cancellation" properties of a balanced modulator.

The author would like to express his sincere gratitude to his advisor, Professor Paul McCollum, for his constant interest and encouragement, from beginning to end of this project.

I would also like to express my gratitude to Mr. Robert Morris, Technical Consultant for the American Broadcasting Company, who suggested the original circuit which was the subject of this investigation. I am also indebted to the American Broadcasting Company, since this thesis topic grew out of work performed on an ABC project.

Appreciation is also expressed to Dr. William L. Hughes, Head, Oklahoma State University, School of Electrical Engineering, whose suggestions on the nonlinear analytic techniques were invaluable.

Last, but certainly not least, the never-failing encouragement and understanding of my wife, Nancy, and our family during these "trying times" is gratefully acknowledged.

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CHAPTER I

INTRODUCTION

Modulation is a part of almost every system which requires the transmission of information from one point to another. As a generalized process, it can be considered as the alteration of some characteristic of the transmission medium to represent the information to be transmitted.

Electrical energy provides a convenient medium for this transmission in many systems. Whether the transmission is through space or guided by conductors it is necessary to "modulate" the electrical signal with the information signal.

The basic electrical signal which is modulated is commonly called the "carrier". In the most general case, this carrier might not be sinusoidal in form. However, regardless of the carrier waveform, if it is periodic, it can be represented by an equivalent Fourier series of sinusoidal terms. Thus, we can consider modulation, in electrical terms, as the alteration of some characteristic of a sinusoidal electrical signal--and understand that, if the need arises, each term of a series must be properly treated. In the same way, we can consider the modulating signal--the information--to be sinusoidal in form.

The characteristics of any specific electrical signal, then, can be described by the basic parameters of a sine (or cosine) waveform. Three parameters serve to completely define such a signal: the amplitude, the frequency, and the phase. Modulation systems are classified in three categories: amplitude modulation, frequency modulation, and phase modulation--depending on which of the basic parameters is varied. However, it is possible to have modulation which varies more than one of the parameters of the carrier.

Television, as a communication system, is capable of transmitting a vast amount of information in any given time interval. Because of this large information handling capacity, the system places some stringent requirements on the modulation process. The color television system, which handles even more information than the monochrome (black-and-white) system, is even more critical. This investigation will be concerned with some of the problems in such TV modulation systems. In the generation of the composite video signal, amplitude modulation is used, so only this type of modulation will be considered.

For sinusoidal carrier and sinusoidal modulation--to which all other cases can be reduced--the output of an amplitude modulation system would appear as in Figure 1.

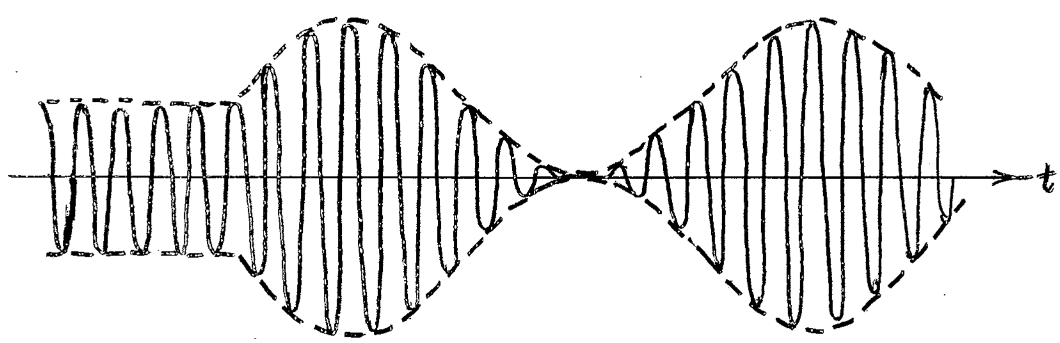

Figure 1. Amplitude Modulation Signal

In this signal, the variation of the amplitude of the carrier has an "envelope" which is the modulation signal.

Analysis of an amplitude modulated signal shows that such a process creates two new signals. They are equally spaced, in frequency, above and below the carrier frequency by the amount of the modulation frequency, and vary in amplitude according to the amplitude of the modulation. In this process the carrier signal is left unaltered in amplitude or frequency. Thus, we might visualize the process as in Figure 2, where the "varying amplitude" and the "frequency-spectrum" representations are depicted for the same signal.

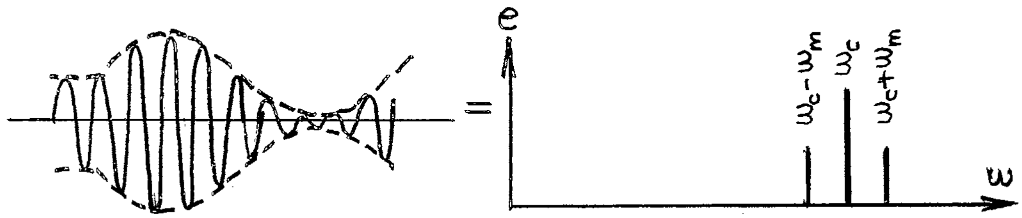

Figure 2. Amplitude-variation and Frequency-spectrum Representation of Amplitude-modulated Signal

It is possible to use this property of amplitude modulation--that the carrier remains unaltered--to advantage. Since the carrier is not changed by modulation, it cannot contain any of the information. Therefore, it is unnecessary to transmit the carrier. If it can be removed without affecting the other two signals, the information-handling capacity of the system will not be affected.

The two signals which are created in the process of amplitude modulation are called "sidebands". A system which transmits only these sidebands is called "suppressed carrier amplitude modulation". This mode of transmission is used to transmit the color information in a color video signal.

Suppressed-carrier AM (amplitude modulation), which does not require the carrier for <u>transmission</u> of the information, <u>does</u> require the carrier at the receiving end of the system, if the information is to be recovered from the sidebands. The carrier must be reinserted into the signal in its original position, if the information is to be recovered by a process known as "demodulation". The requirements for this reinserted carrier are very stringent. It must be the exact frequency and phase of the original carrier, or the recovered signal will be a badly distorted version of the original.

In the color television system, the information necessary to "re-create" the carrier for the demodulation process is handled separately, in time, from the picture information. There is a period during the scanning process--the "sync" interval--in which no picture information is transmitted. This time is used, in part, to transmit a "reference" signal from which the carrier can be generated. This reference signal is called the color burst. Figure 3 depicts how it is transmitted as part of the composite color video signal.

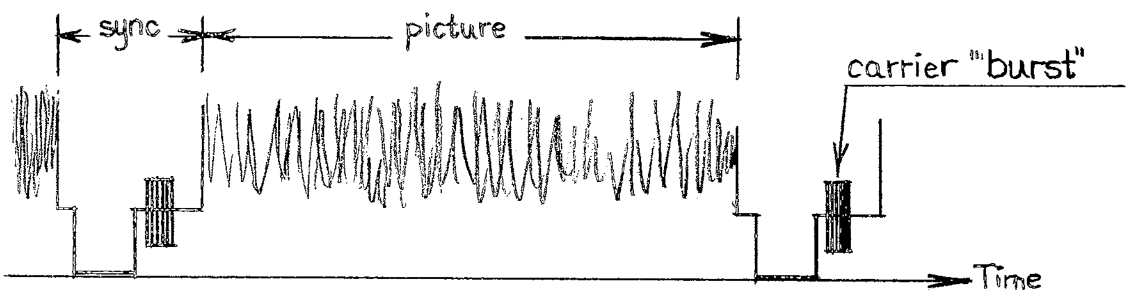

Figure 3. Color Television Video Signal
with Carrier Reference Burst

Since the carrier information is transmitted separately from the actual color information, the presence of any carrier "leak-through" in the supposedly suppressed-carrier signal is destructive. Such a signal appears to be part of the information signal, and after demodulation, serves to distort the information from its original form. Thus, if the suppressed-carrier system of transmission is used, it is necessary that the carrier be truly suppressed.

If low-frequency information signals are to be handled by the system--which is a requirement in TV--the sidebands generated are very close in frequency to the carrier. Because of this proximity it is almost impossible to suppress the carrier by selective filtering. Normally, the only successful technique for removal of the carrier is cancellation by the addition of another signal exactly out of phase and equal in amplitude to the existing carrier signal. This process can be accomplished in a "balanced modulator", which is actually two modulators connected together in such a way that the carrier signals from them cancel, and the sideband signals add.

This investigation will be concerned with a circuit suggested by Mr. Robert Morris, which seems to offer considerable promise as such a balanced modulator. The circuit will be investigated for two of the prime characteristics of such a balanced modulator: the efficiency of generation of sideband signals, and the degree of suppression of the carrier attainable with reasonable operation and adjustment procedures.

CHAPTER II

THEORY OF MODULATION

Amplitude Modulation

The basic carrier signal can be represented mathematically in the form

$$e_c = E_c \cos(\omega_c t+\phi), \qquad [1]$$

where the subscript c is used to identify carrier parameters. Similarly, the information signal can be represented in the form

$$e_m = E_m \cos(\omega_m t+\theta), \qquad [2]$$

with the subscript m identifying modulation parameters. Since only amplitude modulation is to be considered, the time-reference will be selected so that the phase angles ϕ and θ are zero, and the basic signals then become

$$e_c = E_c \cos(\omega_c t) \qquad [3]$$

and

$$e_m = E_m \cos(\omega_m t). \qquad [4]$$

Choice of the cosine signal has not reduced the generality of the representation since, if necessary, a phase angle can be introduced to remove this restriction.

In the process of amplitude modulation, a restriction is placed upon the process so that a reference-level of carrier is transmitted when no modulation signal is present. Also, the change in amplitude of the carrier caused by the modulating signal is restricted to avoid a negative carrier--which infers a 180° phase shift when this occurs. Mathematically, these restrictions are stated by expressing that the amplitude variations of the carrier are represented by

$$1 + k_m E_m \cos(\omega_m t), \qquad [5]$$

where k_m is a constant such that

$$k_m E_{m(max.)} = 1. \qquad [6]$$

With k_m set in value as in [6], it is evident that the "amplitude-variation", as stated at [5], will vary from a maximum of 2 to a minimum of 0 as the modulation signal goes through one complete cycle, at its maximum amplitude.

Introducing the amplitude-variation term at [5] so that it multiplies the amplitude of the carrier signal at [3] yields

$$e_c = E_c\,(1 + k_m E_m \cos\omega_m t)\cos\omega_c t,$$

or,

$$e_c = E_c \cos\omega_c t + k_m E_m E_c \cos\omega_m t \cos\omega_c t. \qquad [7]$$

By aid of the trigonometric identity

$$\cos\alpha \cos\beta = 1/2\,[\cos(\alpha + \beta) + \cos(\alpha - \beta)],$$

[7] can be converted into a form with more physical meaning

$$e_c = E_c \cos\omega_c t \qquad [8]$$

$$+ \frac{k_m E_m E_c}{2}\cos(\omega_c + \omega_m)t \qquad + \frac{k_m E_m E_c}{2}\cos(\omega_c - \omega_m)t.$$

In [8] there is present an unaltered carrier term, and two additional terms, indicating that the process of amplitude modulation has created two new signals. These signals are at frequencies above and below the carrier frequency, spaced from it by the frequency of the modulation. These signals are the sidebands, and they constitute all of the information-bearing portion of the amplitude modulated signal, since the carrier term is unchanged.

Since the multiple-frequency result at [8] was developed purely by mathematical operations, it is logical to question the actual existence of separate signals. It is not difficult, with selective measuring equipment, to show that the signals indicated by [8] actually do exist, that their frequency is that predicted, and that they vary in amplitude as predicted--including that the carrier term is unaltered by modulation.

The investigations of this report are primarily concerned with suppressed-carrier amplitude modulation, so that no further treatment of straight AM will be included here. Introduction at this point has been for the purpose of clarification of the statement that the modulated information is contained solely in the sidebands, and that the carrier is not necessary for the transmission of information by amplitude modulation.

Suppressed-carrier Amplitude Modulation

Removal of the carrier term from the "complete" AM signal is normally accomplished by the addition of an out-of-phase carrier term, with amplitude and phase adjusted so that the sum of the two signals is always zero. In practice, this process is usually performed by a balanced modulator--which is actually two separate modulators--which not only serves to cancel the carrier, but also to increase the amount of sideband signal generated.

One of the two modulators which combine to make such a circuit operates with two input signals exactly as shown at [3] and [4], so that its output is

$$e_c = E_c \cos \omega_c t + \frac{k_m E_m E_c}{2} \cos (\omega_c + \omega_m) t + \frac{k_m E_m E_c}{2} \cos (\omega_c - \omega_m) t. \qquad [9]$$

The second modulator is identical, except that the input signals have been inverted from those at [3] and [4], so that

$$e_c' = -E_c \cos \omega_c t \qquad [10]$$

and

$$e_m' = -E_m \cos \omega_m t. \qquad [11]$$

With these inputs, the output of the modulator, by a process identical to that by which [8] was developed, is

$$e_c' = -E_c \cos \omega_c t + \frac{k_m E_m E_c}{2} \cos (\omega_c + \omega_m) t + \frac{k_m E_m E_c}{2} \cos (\omega_c - \omega_m) t. \qquad [12]$$

With the outputs of the two modulators connected so that the signals at [9] and [12] add, the combined output of the balanced modulator becomes

$$e_o = k_m E_m E_c \cos (\omega_c + \omega_m)t + k_m E_m E_c \cos (\omega_c - \omega_m)t. \qquad [13]$$

In [13], it is apparent that the desired cancellation of the carrier term has taken place. It should be noted that the amplitude of the sidebands generated by the balanced modulator is twice that for a single amplitude modulator, as shown at [9] or [12].

Thoughtful consideration of the two terms remaining as the output of a suppressed-carrier amplitude modulation system leads to an interesting conclusion--either sideband, by itself, contains <u>all</u> of the necessary information about the modulation signal, i.e., its frequency and amplitude. Thus it would appear that it is necessary to transmit only one of them to convey all of the information. On the basis of this conclusion, it is possible to have two suppressed-carrier modulation systems--double sideband, or single sideband. Both of these systems are practical and are in use in many different types of communications systems. Single sideband has the unique advantage that all of the information can be transmitted using half of the frequency-spectrum (band-width) required by the double sideband or full AM system.

In either of the suppressed-carrier systems it is necessary that the carrier be recreated at the receiving end, if the modulation information is to be recovered. The carrier requirements are equally stringent for either system if undistorted reproduction of the original information is to be achieved. Full AM, with the carrier transmitted, possesses the advantage that it is not necessary to recreate the carrier--it is already present as an inherent part of the received signal.

Frequency Changing and Demodulation

In the derivation of the results of amplitude modulation, no restriction has been placed on the relative values of the frequencies of the carrier and modulation. If the sideband products that are created by the process are considered from the viewpoint of being "sum" and "difference" frequencies of the two input signals, some interesting conclusions result--it is possible to "change" the frequency of a signal to any other desired frequency by the process of amplitude modulation, with proper choice of a second input signal to the modulator. This process of frequency-changing is commonly known as "heterodyning", and forms the basis for many useful communications circuits.

One interesting application of this process is in demodulation. Consider as one input to the "frequency-changer" the single-sideband signal previously discussed--for instance,

$$e_1 = k_m E_m E_c \cos(\omega_c + \omega_m)t. \qquad [14]$$

For the second input, use the "recreated" carrier required for demodulation,

$$e_2 = E_c \cos \omega_c t. \qquad [15]$$

With a balanced modulator, and these two input signals, the output signals will appear on two frequencies--one the sum, the other the difference. That is, they will be

$$(\omega_c + \omega_m) + \omega_c = 2\omega_c + \omega_m$$

and

$$(\omega_c + \omega_m) - \omega_c = \omega_m. \qquad [16]$$

The first signal is of little interest and is a spurious product of the process. But, the second signal, [16], is the original modulation. Thus, by the process of modulation, demodulation has been achieved. This process is, in fact, a practical means of demodulation, and is referred to as "product-detection".

CHAPTER III

MODULATION DEVICES AND CIRCUITS

General Considerations

In the consideration of general modulation theory, several assumptions were made, with little--if any--justification or support. Perhaps the most fundamental of these is related to the statement made in developing the basic equation representing an amplitude modulated signal, namely: "Introducing the amplitude-variation term at [5] so that it <u>multiplies</u> the amplitude of the carrier signal------". In this statement, the word multiplies has been underlined to emphasize its importance. The implications of multiplication of one electrical signal by another are far-reaching, and may not be readily apparent.

An inkling of the consequences of such an assumption is seen in a statement by Mason and Zimmermann [1]:

> In a linear system superposition applies; the output due to two simultaneously applied inputs is the sum of the outputs due to each input acting alone. To say the same thing another way, the component response of the system to each input signal is <u>independent</u> of the presence of other inputs.

The underlining is not that of the authors of this statement, but is introduced for emphasis.

Since multiplication of one electrical signal by another is certainly a <u>dependency</u> between them, the statement above infers that multiplication (i.e., modulation) <u>cannot</u> occur in a linear system. Modulation, then, is inherently a <u>nonlinear</u> process, requiring a nonlinear electrical device or circuit. The analysis of such circuits in a mathematically-concise "closed form" is usually not possible. At its best, the result is, hopefully, a close approximation whose validity can only be determined by experimental verification.

The nonlinear devices used to achieve modulation take many forms--inductors, capacitors, resistors, vacuum tubes, semiconductor diodes, or transistors. Any such device is characterized by a fundamental parameter--inductance, capacitance, resistance--which does not remain constant, but is a variable function of the voltage or current of the device.

Nonlinear Circuit Elements

To obtain an understanding of how multiplication (and thus modulation) occurs when a nonlinear circuit element is involved, consider the general mathematical statement of the voltage-current relationship for a resistance--the simplest electrical element--namely,

$$E = F(I), \tag{17}$$

where F(I) is the arbitrary statement which is the characteristic of the device. In the restricted case of <u>linear</u> circuit theory, this relationship reduces to

$$E = IR, \tag{18}$$

where R is a constant not dependent on I.

The relationship at [17] could equally well be stated inversely

$$I = f(E). \tag{19}$$

For any given nonlinear resistance, the functional relationship, F(I) or f(E), may not be known in concise mathematical form. Most frequently, this functional relationship is known only from experimental data, and is presented graphically as a "characteristic-curve" relating the current and voltage.

However, whether this relationship is known mathematically or experimentally, with appropriate restrictions, it may be expressed in the form

$$E = A + BI + CI^2 + DI^3 + EI^4 + \cdots, \qquad [20]$$

or, alternatively

$$I = a + bE - cE^2 + dE^3 + eE^4 + \cdots \qquad [21]$$

With the E-I relationship expressed as at [21], the results of the application of two simultaneous electrical signals can be investigated. Representing the applied voltage as

$$E = E_m \cos \omega_m t + E_c \cos \omega_c t, \qquad [22]$$

substituting [22] into [21] gives

$$\begin{aligned} I = a &+ b[E_m \cos \omega_m t + E_c \cos \omega_c t] \\ &+ c[E_m \cos \omega_m t + E_c \cos \omega_c t]^2 \\ &+ d[E_m \cos \omega_m t + E_c \cos \omega_c t]^3 \\ &+ \cdots . \end{aligned} \qquad [23]$$

Expanding the terms of [23] yields

$$\begin{aligned} I = a &+ b\ E_m \cos \omega_m t + b\ E_c \cos \omega_c t \\ &+ c\ E_m{}^2 \cos^2 \omega_m t + c\ E_c{}^2 \cos^2 \omega_c t \\ &+ 2c\ E_m\ E_c \cos \omega_m t \cos \omega_c t \\ &+ d\ E_m{}^3 \cos^3 \omega_m t + d\ E_c{}^3 \cos^3 \omega_c t \\ &+ 3d\ E_m{}^2\ E_c \cos^2 \omega_m t \cos \omega_c t \\ &+ 3d\ E_m\ E_c{}^2 \cos \omega_m t \cos^2 \omega_c t + \cdots\cdots . \end{aligned} \qquad [24]$$

The terms in [24] involving the square, cube, and product of cosine functions has little physical meaning. By aid of the trigonometric identities:

$$\begin{aligned} \cos^2\alpha &= 1/2[\cos 2\alpha + 1], \\ \cos^3\alpha &= 1/4[\cos 3\alpha + 3 \cos\alpha], \\ \cos\alpha \cos^2\beta &= 1/4[2 \cos\alpha + \cos(\alpha - 2\beta) + \cos(\alpha + 2\beta)], \end{aligned}$$

[24] may be written in the form

$$\begin{aligned} I = a &+ b\ E_m \cos \omega_m t + b\ E_c \cos \omega_c t \\ &+ \frac{c}{2} E_m{}^2 [1 + \cos 2\omega_m t] + \frac{c}{2} E_c{}^2 [1 + \cos 2\omega_c t] \\ &+ c\ E_m\ E_c \cos(\omega_c + \omega_m)t + c\ E_m\ E_c \cos(\omega_c - \omega_m)t \\ &+ \frac{d}{4} E_m{}^3 [3 \cos \omega_m t + \cos 3\omega_m t] + \frac{d}{4} E_c{}^3 [3 \cos \omega_c t + \cos 3\omega_c t] \\ &+ \frac{3d}{2} E_m{}^2\ E_c \cos \omega_c t + \frac{3d}{2} E_m\ E_c{}^2 \cos \omega_m t \\ &+ \frac{3d}{4} E_m{}^2\ E_c \cos(\omega_c - 2\omega_m)t + \frac{3d}{4} E_m{}^2\ E_c \cos(\omega_c + 2\omega_m t) \\ &+ \frac{3d}{4} E_m\ E_c{}^2 \cos(2\omega_c - \omega_m)t + \frac{3d}{4} E_m\ E_c{}^2 \cos(2\omega_c + \omega_m)t + \cdots\cdots . \end{aligned} \qquad [25]$$

Collecting the terms of [25] as coefficients of the various cosine functions yields

$$
\begin{aligned}
I = {} & a + \frac{c}{2} E_m^2 + \frac{c}{2} E_c^2 \\
& + [b\ E_m + \frac{3d}{4} E_m^3 + \frac{3d}{2} E_m\ E_c^2] \cos \omega_m t \\
& + \frac{c}{2} E_m^2 \cos 2\omega_m t + \frac{d}{4} E_m^3 \cos 3\omega_m t \\
& + [b\ E_c + \frac{3d}{4} E_c^3 + \frac{3d}{2} E_m^2\ E_c] \cos \omega_c t \\
& + \frac{c}{2} E_c^2 \cos 2\omega_c t + \frac{d}{4} E_c^3 \cos 3\omega_c t \\
& + c\ E_m\ E_c\ [\cos (\omega_c - \omega_m)t + \cos(\omega_c + \omega_m)t] \\
& + \frac{3d}{4} E_m^2\ E_c[\cos (\omega_c - 2\omega_m)t + \cos (\omega_c + 2\omega_m)t] \\
& + \frac{3d}{4} E_m\ E_c^2[\cos (2\omega_c - \omega_m)t + \cos(2\omega_c + \omega_m)t] \\
& + \cdots\ . \qquad [26]
\end{aligned}
$$

From [26] it is apparent that the application of two sinusoidal voltages to a nonlinear resistance results in the creation of many new signals. These new signals are related to the original voltages by the sums, differences and harmonics of their frequencies. Of particular interest is the pair of terms at the two frequencies $(\omega_c + \omega_m)$ and $(\omega_c - \omega_m)$, since they represent the sidebands encountered in the general theory of modulation. From the presence of these two terms, it is apparent that the application of two sinusoidal signals to a nonlinear circuit element does produce the normal products of modulation.

In the expression for the amplitude modulated signal components at [8], since $k_m E_m = 1$, it is seen that the sideband amplitude is 1/2 that of the carrier. This, of course, is for "full" (100%) modulation, when the maximum amplitude of E_m satisfies the condition just mentioned. An examination of the coefficients of the carrier and sideband components in [26] reveals that their relative magnitudes are determined by the coefficients _b_, _c_, and _d_. The coefficient _d_ introduces some unusual conditions, causing variation of the carrier output amplitude with carrier input amplitude, E_c, _and_ modulation amplitude, E_m. This latter variation, controlled by $E_m^2 E_c$, is _not_ a normal one for amplitude modulation, in which the carrier amplitude is _not_ a function of the modulation signal. Thus, "ideal" AM cannot be obtained from a nonlinear device. In fact, further expansion of the series to higher-order terms in E would show that the coefficients of all odd-order terms would similarly effect the carrier amplitude.

For ideal AM modulation from a nonlinear device, to avoid this variation in E_c with E_m, it would be necessary that all higher-order terms not exist. Under such restrictions, the characteristics of the device would assume the form

$$I = bE + cE^2. \qquad [27]$$

If _b_ and _c_ were related properly to make the sideband amplitude 1/2 that of the carrier term for the maximum allowable value of E_m, full 100% modulation would be attained. However, even under these conditions, the output would contain extraneous terms--the modulation itself, DC components, and second harmonics of both the carrier and modulation. Fortunately, these products are widely separated from the carrier and sidebands in frequency, and can be removed by adequate filters.

Other terms which enter because of the higher-order terms in E are perhaps more destructive than the carrier variation outlined above. These terms will cause the generation of second-order sidebands around the carrier, at frequencies of $(\omega_c + 2\omega_m)$, $(\omega_c - 2\omega_m)$, $(\omega_c + 3\omega_m)$, $(\omega_c - 3\omega_m)$, and so on. These products will be close in frequency to the desired carrier and first-order sidebands, and thus cannot be removed by filtering. They will be transmitted along with the desired signals, will be demodulated along with them, and will appear as "harmonic distortion" of the recovered information signal.

Vacuum Tubes as Nonlinear Elements

In one of the earliest published works on the mathematical representation of vacuum tubes, Carson [2] used a "power series" of the form used at [21] for the general nonlinear resistor. However, the plate current in a vacuum triode is a function of both the grid and plate potentials. Carson took account of this second variable by introduction of a voltage term of the form

$$E = (\mu e_c + e_b),$$

where e_c is the instantaneous grid potential, e_b is the instantaneous plate potential (both with reference to the cathode), and μ is a parameter of the tube relating the relative effectiveness of the grid and plate voltages in changing plate current. μ is known as the "amplification factor" of the tube.

In the power-series expansion of a function of two variables by Taylor's Theorem, the partial derivatives of the dependent variable with respect to the two independent variables are encountered in the coefficients of the various terms. In the vacuum tube, these partials appear as derivatives of plate current with respect to both grid and plate potential, second derivatives with respect to the potentials, and so on. Two of these derivatives have been defined as fundamental parameters of the tube. The derivative of the plate current with respect to plate voltage is defined as the "dynamic plate conductance"--or, as more commonly used, the reciprocal of this is defined as the dynamic plate resistance. The first derivative of the plate current with respect to the grid voltage is defined as the dynamic transconductance of the tube. These derivatives appear in the first power terms of the series. The second and third derivatives, which appear in higher-power terms, are interpreted as the rate of change of the two parameters, and so on.

This fundamental analytic treatment of the vacuum tube, as developed by Carson, has become the accepted approach, and is encountered in almost every treatment of the vacuum tube by means of an equivalent mathematical model.

For simplified "linear" analysis, Carson's treatment is still applicable, but the transconductance and plate resistance are considered to remain constant. This says, in effect, that the higher derivatives do not exist, and the power-series representation is reduced to the first-order terms. This "linearization" of the analysis is very effective for small-signal conditions, allows superposition to be used, and simplifies the mathematics involved.

However, this linear simplification <u>cannot</u> be used in the analysis of modulation circuits, since it is the higher-order terms in the series which contribute the "product" terms necessary for the creation of the sideband components.

Carrier Suppression Techniques.

It has been shown that two modulators with the proper input signals can have their outputs combined so as to cause cancellation of the carrier signal. The actual operation of such a circuit using a nonlinear element can be verified by use of the power-series representations of the devices.

For one of the two elements, consider the input voltage to be of the form

$$E = + (E_c \cos \omega_c t + E_m \cos \omega_m t),$$

and for the second element, consider the input voltage to be of the form

$$E = - (E_c \cos \omega_c t + E_m \cos \omega_m t).$$

For the first input (positive), the output from the element is shown at (26). For the second, the output from the device is

$$\begin{aligned} I = {} & a + \frac{c}{2} E_m^2 + \frac{c}{2} E_c^2 \\ & - [b\, E_m + \frac{3d}{4} E_m^3 + \frac{3d}{2} E_m\, E_c^2] \cos \omega_m t + \frac{c}{2} E_m^2 \cos 2\omega_m t - \frac{d}{4} E_m^3 \cos 3\omega_m t \\ & - [b\, E_c + \frac{3d}{4} E_c^3 + \frac{3d}{2} E_m^2\, E_c] \cos \omega_c t + \frac{c}{2} E_c^2 \cos 2\omega_c t - \frac{d}{4} E_c^3 \cos 3\omega_c t \\ & + c\, E_m E_c\, [\cos (\omega_c - \omega_m) t + \cos (\omega_c + \omega_m) t] \\ & - \frac{3d}{4} E_m^2\, E_c\, [\cos (\omega_c - 2\omega_m) t + \cos (\omega_c + 2\omega_m) t] \\ & - \frac{3d}{4} E_m E_c^2\, [\cos (2\omega_c - \omega_m) t + \cos (2\omega_c + \omega_m) t] \\ & + \ldots . \end{aligned} \qquad [29]$$

By comparison, it is seen that the expression at [29] contains the identical terms found in [26], the only difference being the algebraic sign on some of the terms.

Considering the outputs of the two nonlinear elements to be connected together so that their output signals add algebraically, the combined output is obtained by adding [26] and [29]. This yields

$$I = 2\left[a + \frac{c}{2} E_m^2 + \frac{c}{2} E_c^2\right]$$

$$+ c E_m^2 \cos 2\omega_m t + c E_c^2 \cos 2\omega_c t$$

$$+ 2 c E_m E_c \left[\cos (\omega_c - \omega_m)t + \cos (\omega_c + \omega_m)t\right]$$

$$+ \cdots\cdot \qquad [30]$$

In [30], it is seen that the carrier and all odd harmonics cancel each other, but that all even harmonics and the sidebands add. Thus, two identical nonlinear elements, with proper input signals can be combined to form a "balanced modulator" which will provide suppressed-carrier amplitude modulation.

It must be emphasized that the assumption of identical elements infers that all coefficients in the power series - a, b, c, d, ·····-- must be exactly the same for both elements. In actual elements this is seldom achieved, so that complete balance is seldom attained.

Modulation by Sampling with Switching Devices

It is possible to obtain the normal products of modulation (side-bands) by the use of "sampling" techniques.

The sampling device frequently used for this purpose is a diode (either semiconductor or vacuum) which is switched on and off by the carrier signal. The carrier voltage applied is of relatively high amplitude, so that the device is driven either into hard conduction (a low-resistance state) or is highly reverse-biased (a high-resistance state). By proper circuitry, this action can be used to switch the modulation voltage on and off--at a carrier rate. This switching action, in effect, multiplies the signal by 0 or 1, at carrier rate. When two such circuits are connected "back-to-back", the effect is to multiply the modulation signal by +1 or -1.

Analytically, this switching function is a square-wave, symmetrical about the voltage axis, switching between the states +1 and -1. Mathematically, the Fourier-series representation of this switching function is

$$\begin{aligned} E = {} & \frac{2}{\pi} \cos \omega_c t - \frac{1}{\pi} \cos 2\omega_c t \\ & + \frac{2}{3\pi} \cos 3\omega_c t - \frac{1}{2\pi} \cos 4\omega_c t \\ & + \frac{2}{5\pi} \cos 5\omega_c t - \cdots . \end{aligned} \qquad [31]$$

Multiplying this series by the modulation voltage yields

$$\begin{aligned} E = {} & \frac{2}{\pi} E_m \cos \omega_m t \cos \omega_c t - \frac{1}{\pi} E_m \cos \omega_m t \cos 2\omega_c t \\ & + \frac{2}{3\pi} E_m \cos \omega_m t \cos 3\omega_c t - \frac{1}{2\pi} E_m \cos \omega_m t \cos 4\omega_c t \\ & + \frac{2}{5\pi} E_m \cos \omega_m t \cos 5\omega_c t - \cdots . \end{aligned} \qquad [32]$$

By aid of the trigonometric identity for the product of cosine terms, this can be put into a more meaningful form

$$
\begin{aligned}
E = {} & \frac{1}{\pi} E_m \left[\cos\,(\omega_c - \omega_m)t + \cos\,(\omega_c + \omega_m)t\right] \\
& - \frac{1}{2\pi} E_m \left[\cos\,(2\omega_c - \omega_m)t + \cos\,(2\omega_c + \omega_m)t\right] \\
& + \frac{1}{3\pi} E_m \left[\cos\,(3\omega_c - \omega_m)t + \cos\,(3\omega_c + \omega_m)t\right] \\
& - \frac{1}{4\pi} E_m \left[\cos\,(4\omega_c - \omega_m)t + \cos\,(4\omega_c + \omega_m)t\right] \\
& + \ldots\ldots
\end{aligned}
\qquad [33]
$$

In [33], it is apparent that the "sampling" action of the switching function has created the normal sideband products of amplitude modulation. Since this is a symmetrical (balanced) switching function, the carrier does not appear in the output, nor do any of its harmonics. However, sidebands are produced for each of these harmonics, as though they were the carrier.

Since the pairs of sidebands created in this process are separated from other pairs by a relatively large frequency difference--the carrier frequency--it is possible to select any desired pair of sidebands by normal filter techniques.

Switching circuits, analytically at least, seem to provide the ideal approach for the generation of suppressed carrier amplitude modulation. However, a number of assumptions have been made in the analytic treatment which are difficult to achieve in practical circuits.

First, it has been assumed that the switching function generated by the carrier and diodes is a perfect square-wave--absolutely symmetrical, and with zero rise time. Such a square-wave is difficult to generate! Any deviations from this perfect switching function will cause the creation of undesirable product terms in the output.

Second, for perfect performance, it is necessary that all diodes have exactly the same characteristics. And, we have assumed that these characteristics are "ideal"--zero forward resistance, and infinite back resistance. These, too, are difficult to achieve in practical devices.

However, in spite of these practical difficulties, the diode "switching" modulator is an excellent device for the generation of suppressed-carrier amplitude modulated signals.

CHAPTER IV

A TWIN-TRIODE MODULATOR CIRCUIT

General Circuit Features

A circuit suggested by Mr. Robert Morris was selected for theoretical and experimental investigation. This circuit is shown schematically in Figure 4.

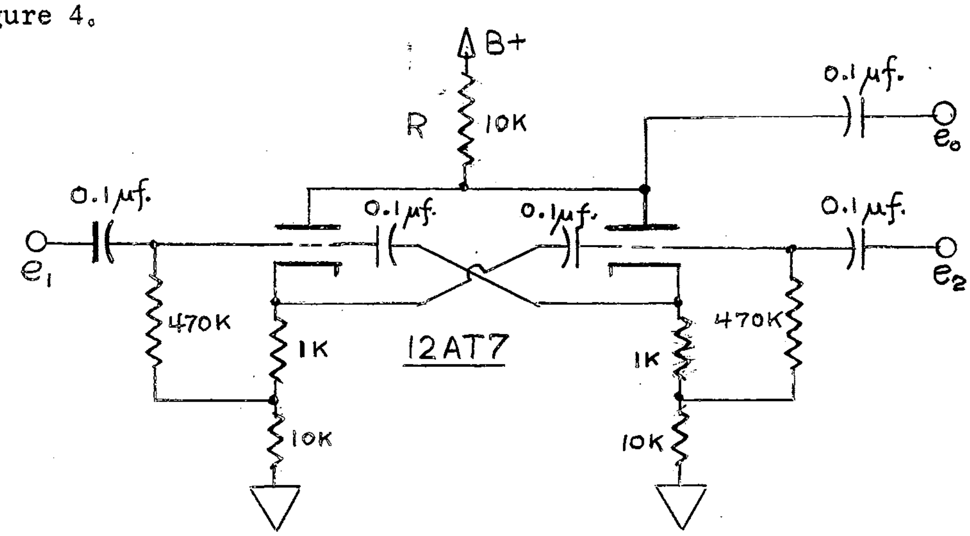

Figure 4. Basic Modulator Schematic Diagram

The circuit seemed to offer some unique advantages as a balanced modulator circuit--if its performance as such a device was adequate. In particular, carrier cancellation was of importance.

The unique advantage of this circuit is that it requires only "single-ended" input signals. All previous discussions have concerned devices which require "push-pull" (double-ended) input signals--that is, a perfectly matched pair of input signals, one positive, the other negative, for <u>both</u> the carrier and modulation. The circuit of Figure 4 does not require these signals for cancellation, but applies the single input voltage to the grid of one tube and to the cathode of the other, simultaneously. The opposing effect on the grid-to-cathode potential of the two tubes increases the plate current in one, and decreases the plate current in the other. With the outputs combined in the common plate-load resistor, R, the two changes cancel each other--thus providing inherent cancellation of the two input signals in the output.

The advantage of this signal-voltage requirement is not immediately apparent, until an investigation is made of the means for generating "push-pull" signals for the conventional balanced modulator. The two most common approaches to this problem are the use of a dual-secondary transformer, or a vacuum-tube "phase-inverter" circuit.

The difficulties of constructing an adequate balanced transformer are almost insurmountable. It is required that the two windings have identical self-inductance, mutual inductance, and stray capacitance. And, if the transformer is to handle a TV video signal as the modulating voltage, this must be true over a frequency range from a few cycles to several megacycles.

The vacuum-tube phase inverter is inherently an unbalanced device, dynamically. If it uses two tubes, the plate current excursions in one will be up while the excursions in the other are down--opposite directions on a "curved" characteristic. Thus, while they may be opposite outputs in phase, they cannot be identical in amplitude because of this "curvature". If a single tube is used in the popular "split-load" phase inverter, one output is taken from the cathode of the tube, the other from the plate. Looking back into this circuit for the equivalent source internal impedances, it is found that they are radically different for the two connections, so that the dynamic performance cannot be identical.

Performance of the circuit of Figure 4 as a modulator will inherently depend on the presence of nonlinear elements--in this case, the two vacuum tubes. The actual circuit has been used as a frequency-conversion heterodyne mixer in practical systems. Performance in these applications has indicated that the nonlinear "mixing" action is very good. Since this is a form of amplitude modulation, the performance of the circuit in this respect was already well established. However, the carrier-rejection capabilities of the circuit were unknown. This investigation will entail a verification of modulator action, and a study of the inherent carrier-cancellation capabilities of the circuit.

A number of "equivalent circuits" for analysis were developed for the actual circuit of Figure 4. It was discovered that the adequacy of these circuits depended entirely on the introduction of input-signal equivalent generators of the correct form. It was only after considerable experimental work was done, indicating the importance of the generator internal impedance, that it was possible to arrive at an equivalent circuit which provided a realistic account of actual circuit performance. The final equivalent circuit used in the analysis is shown in Figure 5.

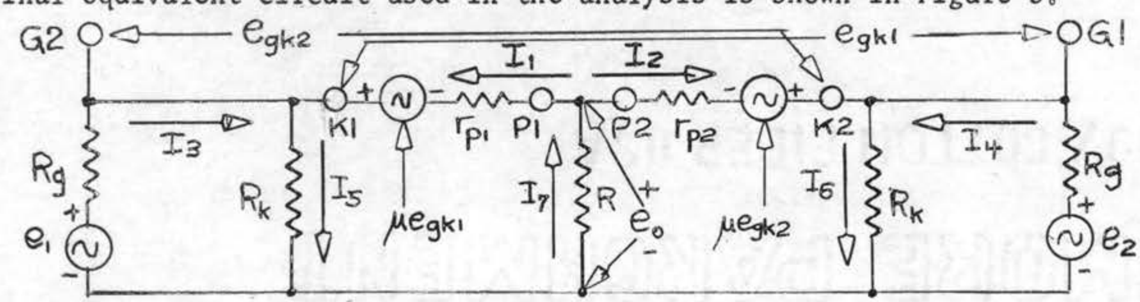

Figure 5. Linear Equivalent Circuit of Modulator

In this equivalent circuit, the grid return resistors have been omitted, since they are some 50 times greater than any other resistance present in the circuit.

It should be noted that this circuit is shown with the usual linear (small-signal) equivalent voltage generator, μe_{gk}, in series with internal impedance, r_p, representing the tube. In the light of previous contentions that modulation is inherently a nonlinear process, and that linear analysis cannot tell anything about modulation--this would seem to be foolhardy.

Several attempts were made to approach the analysis of this circuit by the conventional techniques--replacing each tube, analytically, by the power-series representation of a nonlinear element. However, in every case, the algebra encountered in the process of analysis became so complicated as to defy further progress. In most cases, it was required to solve two simultaneous equations in two unknowns--with each equation containing cubic or quartic terms in each of the two unknowns, and cross-products between them. Attempts to simplify these equations failed because simplifications could not be made because physical meaning was obscure.

It was felt that linear analysis, which was not too difficult, should give some meaningful results for the inherent carrier-cancellation capabilities of the circuit. Direct carrier terms appear in the output primarily from first-order terms, in the power-series representation. Thus, linear analysis, using just the first-order terms, should be indicative of the carrier-cancellation possibilities.

Dr. William L. Hughes suggested that nonlinear analysis might be successfully accomplished by the introduction of nonlinearities into appropriate terms in the results of the linear analysis. With this approach it would be possible to keep identification of various terms with their physical meaning. If simplifications were required, they could be based on assumptions about individual terms for which physical interpretation was not completely obscure.

Linear Analysis

Linear analysis of the circuit was performed by conventional methods. Current-node and voltage-loop equations were written to obtain a set of seven independent equations in seven unknowns--the seven currents indicated in Figure 5. Two auxiliary equations including the voltages e_{gk1} and e_{gk2} were written so that these two voltages could be eliminated in the equations which involved the equivalent generators, μe_{gk1} and μe_{gk2}.

The seven independent equations were solved for I_1 and I_2, which are, in reality, the plate currents of the two tubes. An expression for the output voltage was obtained from these, since their sum passes through the common plate-load resistor, R. This final output-voltage expression is

$$e_o = \frac{bR}{R_2^2 - (r_{p1} + R_1)(r_{p2} + R_1)} \Big[[(\mu+1)r_{p2} - \mu r_{p1} + (2\mu+1)bRg]e_1 + [(\mu+1)r_{p1} - \mu r_{p2} + (2\mu+1)bRg]e_2 \Big], \quad [34]$$

where

$$R_1 = R + (\mu+1)bRg, \quad R_2 = R - \mu bRg, \quad \text{and} \quad b = \frac{R_k}{R_g + R_k}. \quad [35]$$

In this expression for the output voltage, the assumption has been made that the μ of the tubes is the same. It is felt that this is a realistic assumption, since the amplification factor of a tube is its most constant basic parameter.

It is apparent from [34] that, even with the assumption of identical tubes ($\mu_1 = \mu_2$, and $r_{p1} = r_{p2}$), there is some component of the input voltages present in the output, indicating that carrier-cancellation is not inherently complete in this circuit.

However, in the experimental model the circuit could be adjusted so that carrier cancellation was good. In order to ascertain what conditions in the circuit would make this true, the coefficient of one of the input voltage terms was equated to zero. Assuming the carrier is represented by the voltage generator e_1, equating the coefficient of the e_1 term in [34] to zero yields

$$(\mu + 1) r_{p2} - \mu r_{p1} + (2\mu + 1) \frac{R_g R_k}{R_g + R_k} = 0,$$

or,

$$\mu r_{p1} = (\mu + 1)\, r_{p2} + (2\mu + 1) \frac{R_g R_k}{R_g + R_k};$$

from which

$$r_{p1} = \frac{\mu + 1}{\mu}\, r_{p2} + \frac{2\mu + 1}{\mu} \frac{R_g R_k}{R_g + R_k}. \qquad [36]$$

Thus, in [36], on the basis of a linear analysis, we have stated the requirement for a basic <u>unbalance</u> in the circuit for carrier cancellation to occur.

If the μ of the tubes is high, the first term will contribute little to this required unbalance. However, it is observed that the second term involves both tube parameters and circuit components. If μ is high, the coefficient of this term is approximately 2. The circuit components appear as the parallel-equivalent resistance of the generator internal resistance, R_g, with the cathode resistor, R_k. Since R_k is relatively high, this requires that R_g should be quite low, if the contribution of this second term to the required unbalance is to be minimized.

The implications of this requirement are not apparent unless a slightly different outlook on the basic circuit is used. Upon inspection of the circuit, it is found that a positive-feedback loop exists around the entire circuit--from the grid of V1 to the cathode of V1, thence to the grid of V2, to the cathode of V2, and then back to the grid of V1. The feedback around this loop is positive, but since the input-output for each tube is grid-cathode, the total gain is appreciably less than one. However, even though the loop gain has no possibility of causing self-oscillation, it should have an appreciable regenerative effect. From this viewpoint, the requirement that R_g must be low is, in effect, a requirement that this feedback loop be "broken".

Nonlinear Analysis

The linear analysis of the preceding section will be extended to a nonlinear analysis by the "perturbation" of appropriate parameters of the tubes.

From the fundamental definition of the μ of a tube,

$$\mu = \frac{\partial e_b}{\partial e_c}\bigg|_{(i_b \text{ constant})}, \qquad [37]$$

theoretically at least, there is no variation of the μ with changes in current. An examination of the characteristics of an actual triode shows that over a quite wide range of current this is essentially true. Only near cutoff (very low current) does the μ of a practical triode change appreciably. On this basis, the linear analysis will not be perturbed for changes in μ.

An examination of the relationship between r_p and current in a triode reveals that, theoretically, the plate resistance varies inversely with the cube-root of the current in the tube [3]. An examination of the actual characteristics of a practical triode as shown in Figure 6, reveals that, at least in form, this variation is approximately correct.

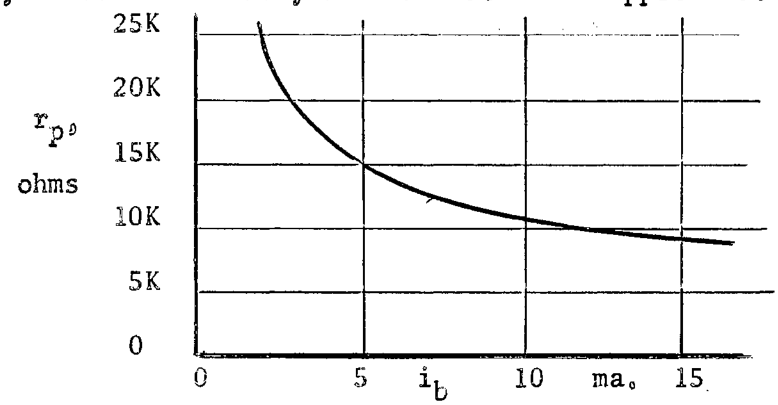

Figure 6. r_b - i_b Relationship for Typical Triode

At very low current the plate resistance is very high, and, as the plate current increases, the plate resistance drops appreciably. On this basis it will be assumed that the plate resistance, r_p, varies inversely with plate current, as a "perturbed" value. Specifically, the variation is assumed to be

$$r_{p1} = r - k\, I_1, \qquad [38]$$

and

$$r_{p2} = r - k\, I_2, \qquad [39]$$

where $\underline{r}$ is the plate resistance at the quiescent-point of operation (no-signal), and k is the slope of the r_p - i_b curve at that point. Since the r_p - i_b curve is not a straight line of negative slope, this assumption is only an approximation. However, it does give a reasonable approximation over a fair range of tube current. With the assumption of any other form of variation, higher-order terms enter into the analysis, and the same difficulty is encountered as in the full nonlinear analysis.

In the linear analysis, the expressions obtained for I_1 and I_2 are

$$I_1 = \frac{\mu_2 bR_2 + (\mu_1 + 1)\, b\, (r_{p2} + R_3)}{R_2R_4 - (r_{p1} + R_1)\,(r_{p2} + R_3)}\, e_1 - \frac{(\mu_2+1)bR_4 + \mu_1 b\,(r_{p2} + R_3)}{R_2R_4 - (r_{p1} + R_1)(r_{p2} + R_3)}\, e_2, \qquad [40]$$

and

$$I_2 = \frac{\mu_1 bR_4 + (\mu_2 + 1)\, b\, (r_{p1} + R_1)}{R_2R_4 - (r_{p1} + R_1)\,(r_{p2} + R_3)}\, e_2 - \frac{(\mu_1+1)bR_4 + \mu_2 b\,(r_{p1} + R_1)}{R_2R_4 - (r_{p1} + R_1)(r_{p2} + R_3)}\, e_1, \qquad [41]$$

where R_1, R_2, R_3, R_4, and b are constants involving the μ of the tubes and circuit components, introduced only for algebraic convenience.

The necessity for a simplifying assumption arises at this point. When I_1 changes, r_{p1} changes. But, since [40] involves r_{p1}, changes in r_{p1} cause changes in I_1--an immediate involvement in higher-order terms in both current and r_p. In order to simplify this situation, the assumption is made that the changes in current which cause a change in r_p result only from the applied voltages, e_1 and e_2. Stated mathematically, [40] and [41] assume the form

$$I_1 = k_1e_1 - k_2e_2, \qquad [42]$$

and

$$I_2 = k_1e_2 - k_2e_1, \qquad [43]$$

where the constants k_1 and k_2 are evaluated by using the value of r, as defined at [38] and [39], for r_{p1} and r_{p2}.

Substitution of [42] and [43] into [38] and [39] gives

$$r_{p1} = r - k\,(k_1e_1 - k_2e_2) = r - k_3e_1 + k_4e_2, \qquad [44]$$

$$r_{p2} = r - k\,(k_1e_2 - k_2e_1) = r - k_3e_2 + k_4e_1, \qquad [45]$$

which are the "perturbed" values of r_{p1} and r_{p2} which are to be introduced into the results of the linear analysis.

With [44] and [45] introduced into the expression for e_o at [34], analysis becomes merely an extended exercise in algebra. The final result of this process is

$$\begin{aligned} e_o = \frac{bR}{k_5}\Big[& k_9 e_1 + k_9 e_2 \\ & + (k_{11} - k_9 k_{12})\, e_1^{\,2} + (k_{11} - k_9 k_{12}) e_2^{\,2} \\ & + (k_{10} - 2k_9 k_{12}) e_1 e_2 \\ & + k_{15}\, e_1^{\,2} e_2 + k_{15}\, e_1 e_2^{\,2} \\ & + k_{16}\, e_1^{\,3} + k_{16}\, e_2^{\,3} \\ & + \cdots \Big], \end{aligned} \qquad [46]$$

where the k's are constants dependent on "mean" tube parameters, the rate-of-change of r_p with changes in i_b, and the various circuit components.

It is now convenient to represent e_1 and e_2 as the normal carrier and modulation sinusoids

$$e_1 = E_c \cos \omega_c t, \qquad [47]$$

and

$$e_2 = E_m \cos \omega_m t. \qquad [48]$$

By aid of the trigonometric identities [4]:

$$e_1^2 = \frac{E_c^2}{2}[\cos 2\omega_c t + 1],$$

$$e_2^2 = \frac{E_m^2}{2}[\cos 2\omega_m t + 1],$$

$$e_1 e_2 = \frac{E_c E_m}{2}[\cos(\omega_c - \omega_m)t + \cos(\omega_c + \omega_m)t],$$

$$e_1^3 = \frac{E_c^3}{4}[3\cos\omega_c t + \cos 3\omega_c t],$$

$$e_2^3 = \frac{E_m^3}{4}[3\cos\omega_m t + \cos 3\omega_m t],$$

$$e_1^2 e_2 = \frac{E_c^2 E_m}{4}[2\cos\omega_m t + \cos(2\omega_c - \omega_m)t + \cos(2\omega_c + \omega_m)t],$$

and

$$e_1 e_2^2 = \frac{E_c E_m^2}{4}[2\cos\omega_c t + \cos(\omega_c - 2\omega_m)t + \cos(\omega_c + 2\omega_m)t]. \qquad [49]$$

The final expression for e_o becomes

$$\begin{aligned} e_o = \frac{bR}{k_5}\Bigg[&(k_{11}-k_9k_{12})\left(\frac{E_m^2}{2} + \frac{E_c^2}{2}\right) + \left(k_9E_m - k_{16}\frac{3E_m^3}{4} - k_{15}\frac{E_c^2E_m}{2}\right)\cos\omega_m t \\ &+(k_{11}-k_9k_{12})\frac{E_m^2}{2}\cos 2\omega_m t - k_{16}\frac{E_m^3}{4}\cos 3\omega_m t \\ &+(k_{10}-2k_9k_{12})\frac{E_mE_c}{2}[\cos(\omega_c - \omega_m)t + \cos(\omega_c + \omega_m)t] \\ &-k_{15}\frac{E_m^2E_c}{4}[\cos(\omega_c - 2\omega_m)t + \cos(\omega_c + 2\omega_m)t] \\ &+\left(k_9E_c - k_{16}\frac{3E_c^3}{4} - k_{15}\frac{E_cE_m^2}{2}\right)\cos\omega_c t + (k_{11} - k_9k_{12})\frac{E_c^2}{2}\cos 2\omega_c t \\ &-k_{15}\frac{E_mE_c^2}{4}[\cos(2\omega_c - \omega_m)t + \cos(2\omega_c + \omega_m)t] - k_{16}\frac{E_c^3}{4}\cos 3\omega_c t + \cdots\Bigg]. \end{aligned} \qquad [50]$$

The result of this analysis at [50] contains all the terms that would be expected in the consideration of modulation by a power-series expansion of the characteristics of a nonlinear element.

The coefficients of the various terms assume the normal relationships to amplitudes of the two input signals, as comparison of [50] with [26] will reveal.

By the presence of the carrier term in the output signal, it is apparent that inherently this circuit does not provide carrier cancellation, as the normal balanced modulator does. Furthermore, as a result of the third term in the coefficient for the carrier term, it is apparent that the carrier feed-through is not constant with a constant level of carrier, but also varies as the modulation amplitude, E_m, varies. Even if the circuit is deliberately unbalanced to provide carrier cancellation, as indicated in the linear analysis, the carrier cancellation will not be stable, but will vary with modulation. This is highly undesirable in any suppressed-carrier system.

Although the circuit does not provide complete carrier cancellation, the carrier component in the output is very low. Thus, the coefficient of the carrier term

$$k_9 E_c - k_{16} \frac{3 E_c^3}{4} - k_{15} \frac{E_m^2 E_c}{2}$$

must, in actual numerical evaluation, be a small number relative to the other terms. Thus, k_9, k_{15}, and k_{16} must be small individually.

CHAPTER V

EXPERIMENTAL INVESTIGATION

Preliminary Work

An experimental model of the circuit of Figure 4 was constructed early in the sequence of this investigation. The circuit showed some promise as a balanced modulator. However, it did not inherently balance for carrier cancellation, and no adjustment for exact balance was provided.

A period of "tinkering" followed, in the search for a modification of the original circuit which would allow precise adjustment for carrier cancellation with a minimum number of adjustments and controls. At the same time, general observation of the many circuit variations served to give a better understanding of the performance to be expected under various conditions, and of the relative effect of the different circuit components. This experience proved invaluable in the evolution of the final equivalent circuit shown in Figure 5.

At one time during this period of experimentation, every resistor in the circuit had been replaced by a potentiometer--so that a wide range in control of the operating point and of effective circuit values and their interrelationship could be obtained. One by one, reasonably optimum values for various components were determined, with emphasis on ease and precision of adjustment for carrier cancellation, while maintaining reproducible results with various tubes of the same type.

Simultaneously with this experimental work, the theoretical analysis was developing--with the two analyses complementing each other. As an example of this--when the theoretical analysis indicated that the amount of unbalance between the operating point of the two tubes would seemingly be improved by the use of higher μ tubes, this point was investigated. There exists a series of four tubes, all dual triodes, similar in general characteristics, with different amplification factors. These are:

12AU7 - - - - - - - - - - - - μ = 20,
12AY7 - - - - - - - - - - - - μ = 40,
12AT7 - - - - - - - - - - - - μ = 60,
and 12AX7 - - - - - - - - - - - - μ = 100.

The circuit was altered to accommodate these various tubes, and circuit performance checked. It was, indeed, found that the higher μ tube gave better carrier balance, with closer agreement in the static operating point. At the same time, it was discovered that the balance adjustment became very critical and unstable with the high μ tube, so that a compromise, the 12AT7, was used in the final circuit version.

One major modification was made in the original circuit on the basis of experimental findings. Early in the investigation it was discovered that carrier cancellation was seriously affected by the modulation level. At the same time, it was observed that there was a fairly large shift in the Q-point with modulation level. It was felt that a portion of this Q-point shift could be overcome by the use of fixed-bias on the tube grids, rather than obtaining the operating bias by returning the grid resistor to a tap on the cathode resistor. When tried, this did seem to improve the circuit balance stability, and this mode of bias is included in the final circuit. This also turned out to be an improved way to adjust the circuit for carrier cancellation--all adjustments became d.c. controls, rather than variables in the signal circuits. However, this modification did not eliminate the variation of carrier feed-through with change in modulation--which later turned out to be a basic characteristic of the circuit, when the nonlinear analysis had been completed.

The final version of the circuit which was evaluated theoretically and experimentally is shown in Figure 7.

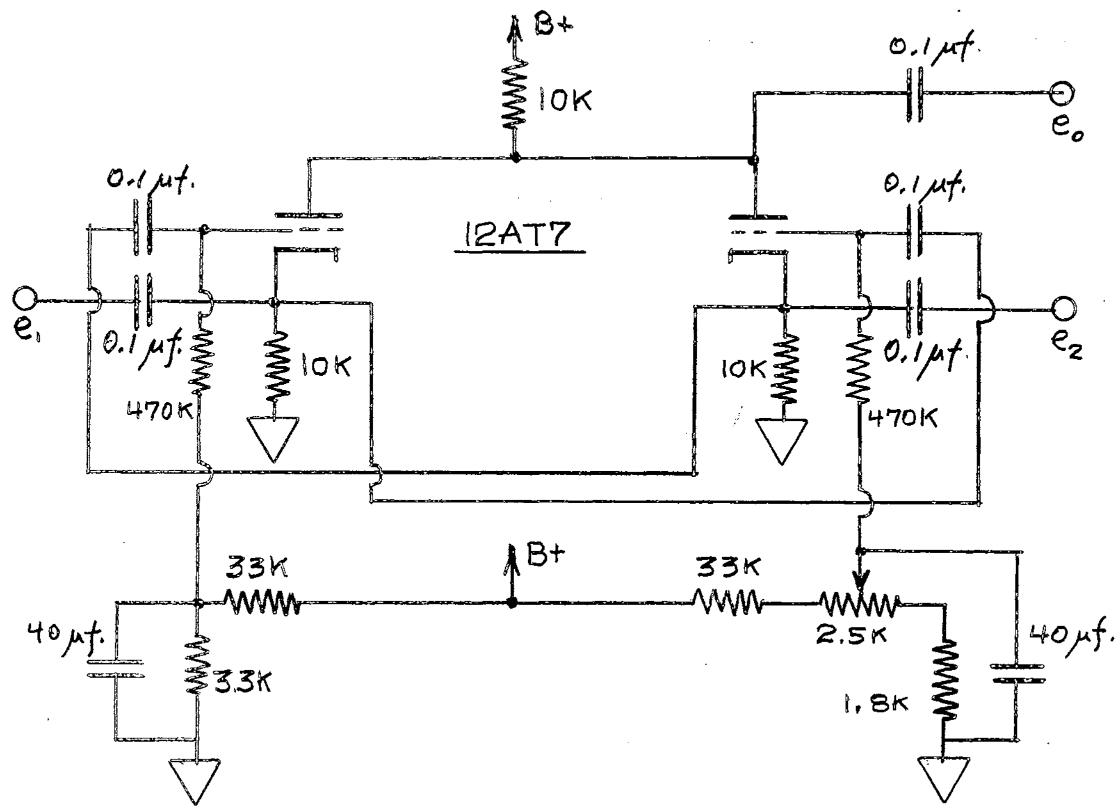

Figure 7. Schematic Diagram of Test Circuit

This circuit was tested extensively after the linear and nonlinear analysis had been completed. The data from these tests is compiled in Appendix C.

Correlation of Experimental and Theoretical Studies

The requirement for carrier balance, as determined in the linear analysis, was generally verified. It was found that a very low value of generator internal impedance was required for carrier cancellation. The signal generator used had an internal impedance of approximately 10 ohms. With the generator connected directly to the input terminals only a minor unbalance of the two tubes was required for carrier cancellation. The addition of resistance as low as 10 ohms in series with the generator required a radical change in the operating point of the adjustable-bias tube, to provide the proper r_p for carrier cancellation. Under such conditions, although the circuit could be balanced statically, the general characteristics of the two tubes were so different that very poor performance was obtained when modulation was applied.

By [36], it is required that the plate resistance of the tube V1 must be larger than that for V2 when the carrier is applied to the grid of V1 and the cathode of V2, if carrier cancellation is to take place. Since r_p varies inversely with plate current, this would require that V1 be operated with a lower current than V2. This condition was experimentally verified (see data groups 1 and 2, page 75, Appendix C). It was also observed that a series resistance in the generator lead required even further unbalance of tube current in this same direction, as would be expected, from [36]. These conditions, while recorded for only one tube, were observed to be true for four different tubes of the same type--with one exception. This one tube required almost equal plate current in the two sections for carrier cancellation. When checked in a mutual-conductance tube checker, this tube was found to exhibit a wide difference in the characteristics of the two sections. The other three tubes were closely balanced between the two sections.

For preliminary signal analysis, the circuit was operated with a carrier frequency of 14 kilocycles per second and a modulation frequency of 240 cycles per second. The choice of these frequencies was determined by the equipment on hand. It was desired to operate the circuit at a carrier frequency such that the third harmonic of the carrier would be less than 50 kilocycles, so that available spectrum-measuring equipment could be used. The exact carrier frequency selected was determined by the availability of a band-pass filter for a pass-band of 12.5-15.7 kilocycles, which could be used to filter the double-sideband signal from the other frequency-components for observation. It was felt that, if preliminary investigations warranted, the circuit could later be tested with video modulation and a carrier frequency of 3.58 megacycles-- as it would be used in color TV balanced-modulator service.

Using the spectrum analyzer, measurements were made for most of the signal components indicated to be present in the output by the theoretical analysis. These measurements are tabulated in Appendix C.

With the final experimental circuit as shown in Figure 6, tube type selected, and circuit components adjusted to final values, the coefficients of the various signal terms in [50] could be numerically evaluated. The final result for e_o, with all constants evaluated, is

$$\begin{aligned} e_o = {} & 3.75\,(E_m^{\,2} + E_c^{\,2}) - 9.09(25\,E_m + 0.64\,E_m^{\,3} + 0.002\,E_mE_c^{\,2})\cos\omega_m t \\ & + 3.75\,E_m^{\,2}\cos 2\omega_m t - 1.88\,E_m^{\,3}\cos 3\omega_m t \\ & - 7.50\,E_mE_c[\cos(\omega_c - \omega_m)t + \cos(\omega_c + \omega_m)t] \\ & - 0.01\,E_mE_c[\cos(\omega_c - 2\omega_m)t + \cos(\omega_c + 2\omega_m)t] \\ & - [0.227\,E_c + 0.0058\,E_c^{\,3} + 0.018\,E_m^{\,2}E_c]\cos\omega_c t \\ & + 3.75\,E_c^{\,2}\cos 2\omega_c t \\ & - 0.009\,E_mE_c^{\,2}[\cos(2\omega_c - \omega_m)t + \cos(2\omega_c + \omega_m)t] \\ & - 0.019\,E_c^{\,3}\cos 3\omega_c t \\ & + \cdots\,. \end{aligned} \qquad [51]$$

Insertion of actual magnitudes of carrier and modulation voltages, E_c and E_m, into this expression will yield the theoretically-predicted magnitudes of the various components. Since measurements were made of all components for $E_c = 1$ volt (rms) and $E_m = 1$ volt (rms), these magnitudes were selected for comparison of theoretical and experimental results.

The terms of greatest interest--the first-order sidebands--with $E_c = E_m = 1$, are predicted to be 7.5 volts. Experimentally, (see data group 3, page 71, Appendix C) it is found that the measured signal on these two frequencies is 0.46 volts. This discrepancy of more than 10 to 1 is disappointing.

The second term of any significance in the output is the second harmonic of the carrier. With the stated input conditions, this is predicted to be 3.75 volts. It was actually measured to be 6.1 volts. The agreement here is greatly improved over that for the carrier term.

Other components were checked, and the comparison of theoretical and experimental values, generally, lies in the range of a discrepancy of from 5 to 10 times. However, one encouraging point is that the relative magnitudes of the various components--one compared to another--is quite reasonable. The only component which was found to be far out of line was the second-order sideband--$(\omega_c + 2\omega_m)$--which is less than ten percent of the value predicted. In fact, with one volt inputs, the predicted value was 0.01 volts, but experimentally it was far lower than this, and could not be measured--which means that it was considerably lower than one millivolt.

For the third harmonic term, which should increase as the cube of the applied carrier voltage amplitude, the amplitude varies approximately as the cube of the applied voltage for only very low values.

At carrier amplitudes of approximately 2 volts or more, the increase "levels off" very rapidly. This would seem to indicate that the analysis has no application for large-signal conditions, and can only be applied under small-signal conditions.

Particular investigation of the carrier-balance condition with changing modulation voltage was made, to attempt a verification of the conditions predicted by the theoretical analysis. It was found that carrier could be balanced to about the same suppression level for any given modulation voltage. However, once balanced for that particular modulation voltage, any change, up or down, in the modulation voltage caused an increase in the carrier feed-through signal. If the carrier voltage is fixed, the theoretical investigation, based on the last term of the coefficient of the carrier signal, predicts that this feed-through should change as the square of the modulation voltage. In practical application, it would be necessary to balance for carrier cancellation with the modulation voltage at zero. In Figure 8, the variation of the carrier feed-through voltage with variation of the modulation voltage is shown, under the condition of carrier-balance with modulation voltage zero.

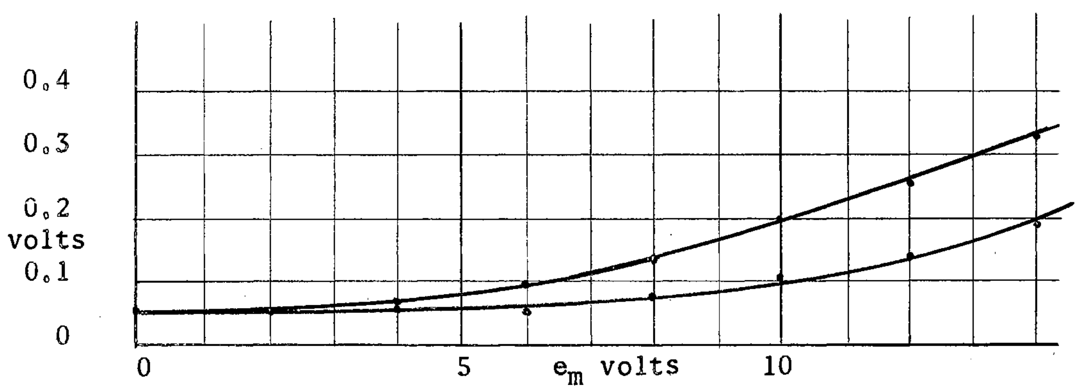

Figure 8. Carrier-balance Shift with Modulation

In Figure 8, it is seen that, although the feed-through of carrier does not increase as the square of the modulation voltage, the relationship is not first-order either--perhaps somewhere between 3/2 and 2.

Under average operating conditions, the carrier suppression could be adjusted to be about 38-40 db. below the side-band signal level at maximum undistorted modulation. If balanced under maximum modulation-voltage conditions, this suppression ratio could be improved to approximately 45 db. at maximum modulation. However, the carrier signal, under these conditions, would rise on the order of 15 db. as modulation voltage was reduced. Since this is an impractical adjustment procedure in any event, it is of little concern--except as an indicator of changing balance conditions in the circuit.

The data obtained was plotted to indicate "linearity" of modulation--a one-to-one correspondence between sideband amplitude and modulation amplitude. This was found to be extremely linear over a very wide range of modulation voltage--0 to 20 volts (rms) in fact.

The sideband amplitude was also plotted against carrier amplitude for various modulation voltages, and found to be quite linear up to approximately 1.5 volts (rms) of carrier. Above that point, the sideband amplitude rapidly "flattened out", and above 2.5 volts, very little increase was found. However, in this "saturated" range for carrier-voltage variations, the modulation-sideband relationship remained linear.

These relationships are shown in Figures 9 and 10.

Circuit Modifications

The limitation of this circuit to extremely low values of generator internal impedance was recognized as a serious problem for any practical

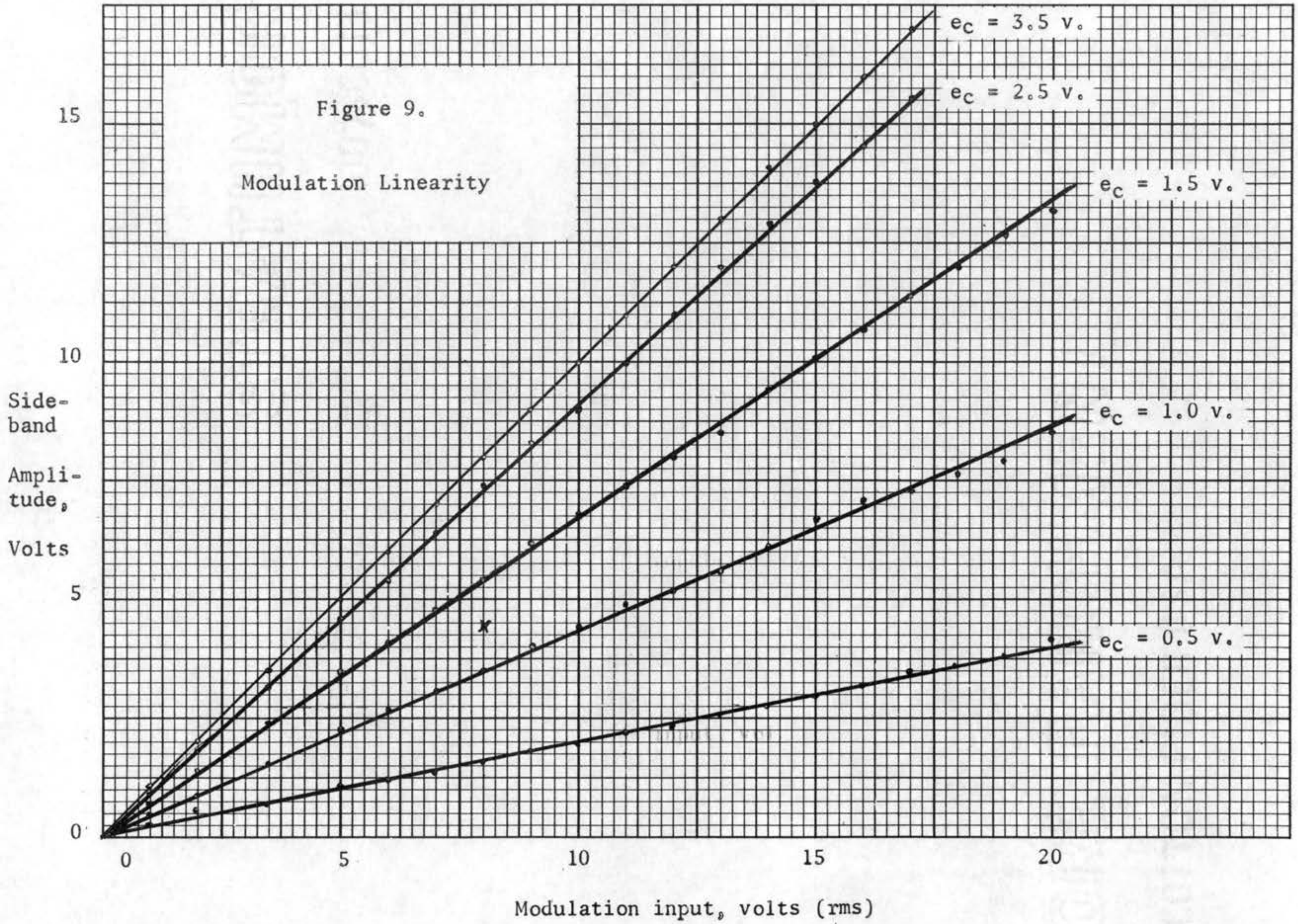
Figure 9.
Modulation Linearity
e_c = 3.5 v.
e_c = 2.5 v.
e_c = 1.5 v.
e_c = 1.0 v.
e_c = 0.5 v.
15
10
5
0
Side-band Amplitude, Volts
0
5
10
15
20
Modulation input, volts (rms)

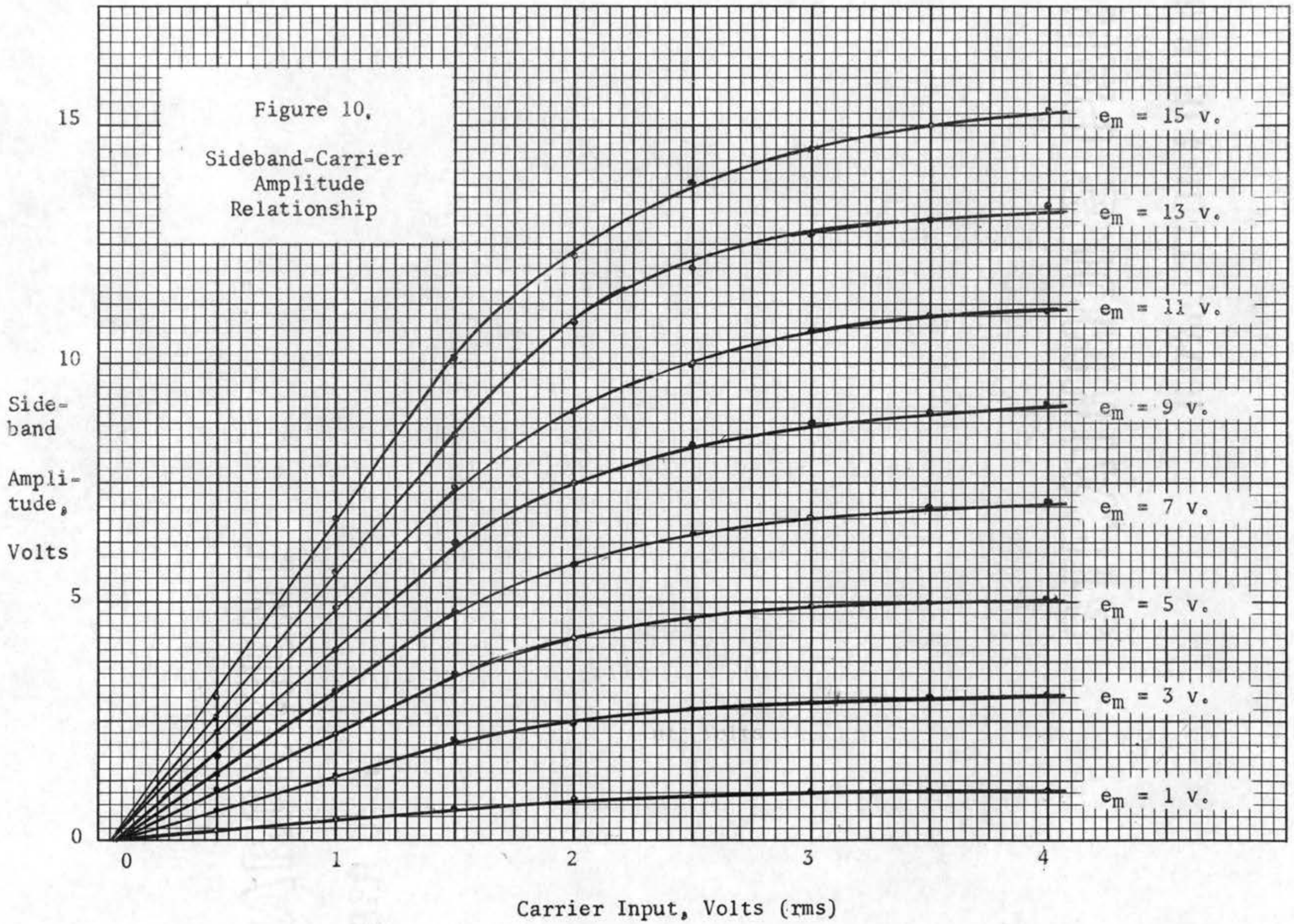
Figure 10.
Sideband-Carrier
Amplitude
Relationship
15
10
5
0
Side-
band
Ampli-
tude,
Volts
e_m = 15 v.
e_m = 13 v.
e_m = 11 v.
e_m = 9 v.
e_m = 7 v.
e_m = 5 v.
e_m = 3 v.
e_m = 1 v.
0
1
2
3
4
Carrier Input, Volts (rms)

use. A modification of the circuit to alleviate this was constructed and tested briefly.

The concept of the modification was to provide "built-in" cathode followers to drive the cathodes of the two tubes in the modulator circuit. Coupling from the cathode followers to the cathodes of the modulators was achieved through the use of a common cathode resistor. This circuit modification is shown in Figure 11.

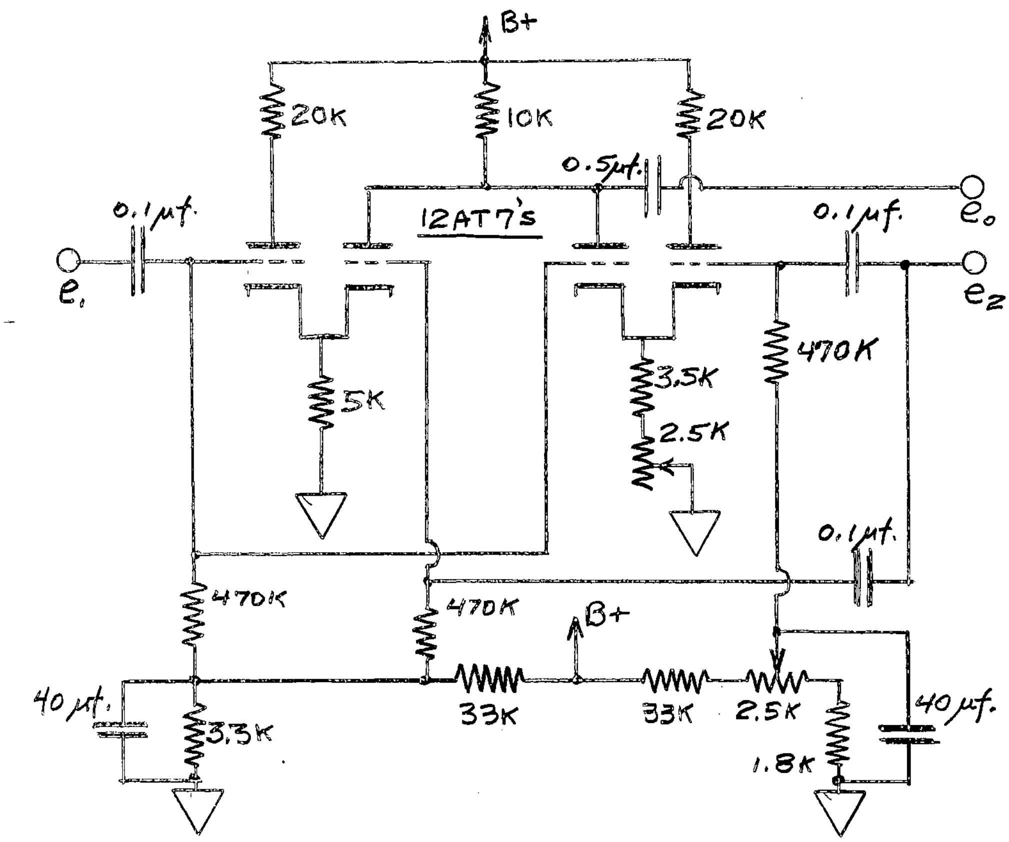

Figure 11. Modified Modulator for High-impedance Signal Sources

The performance of this circuit, in general, was quite similar to that of the circuit of Figure 6. Balance adjustment was very critical, with the controls as shown in Figure 11. In a practical circuit, it would be necessary to provide "vernier" controls on both of the balance adjustments. Once adjusted, the circuit exhibited good stability of balance adjustment. The circuit achieved the desired result of isolating the cathode circuit from any direct effect from the generator impedance. Average carrier suppression was in the range of 35-38 db. down from maximum sideband amplitude. The range of linear modulation was greatly reduced, and the circuit could easily be overloaded.

A second modification of the original circuit was constructed, to investigate the use of pentodes instead of triodes. In the early phases of the theoretical analysis, there was an indication that carrier suppression might be improved by the use of extremely high r_p tubes. This, logically, led to the use of the pentode-type of tube, with its extremely high r_p. On this basis, the circuit of Figure 12 was constructed and tested briefly.

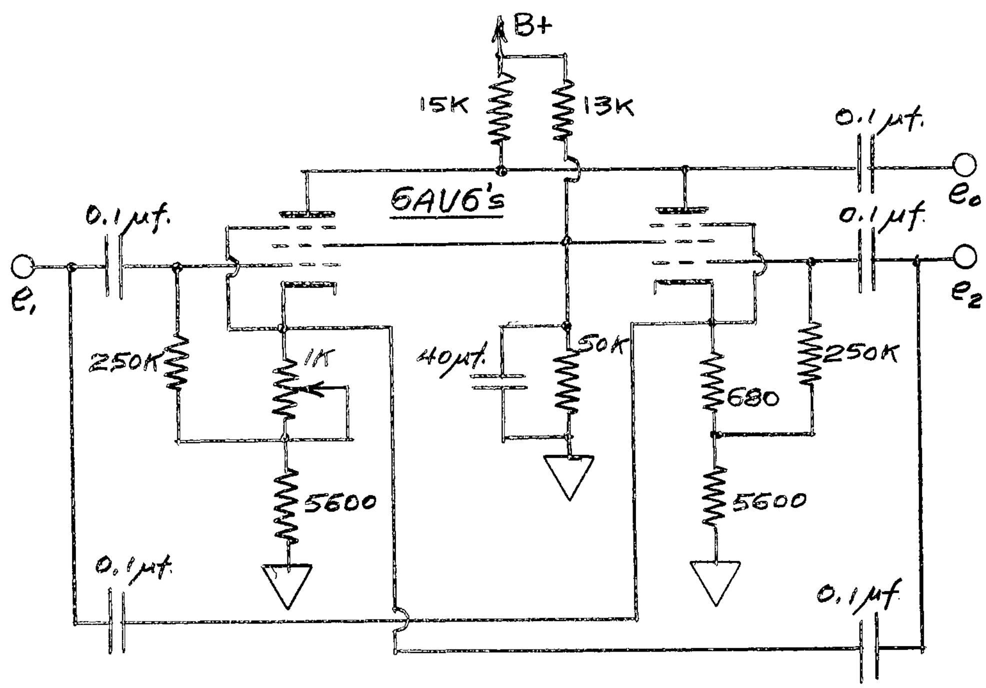

Figure 12. Modified Modulator for Pentodes

General performance of the circuit was much the same as the triode version. Carrier balance did not seem to be improved. At least in the version tested, carrier balance was difficult to attain, and was not stable. On the basis of these indications, the circuit was abandoned, and investigation limited to the triode circuit of Figure 6.

Miscellaneous Observations

The waveforms generated in the output circuit of the test modulator were very interesting. A very large second harmonic component was present--as predicted from the theoretical analysis. There seemed to be very little distortion of the waveform of this second harmonic signal when the circuit was properly balanced for carrier suppression. This condition is verified in the analysis, where harmonics above the second are found to be of very small amplitude.

Upon the application of modulation voltage, alternate cycles of this second harmonic signal were increased, and the remaining alternate cycles were decreased. The "envelope" of these increased and decreased cycles followed the form of the modulation voltage. This variation in the amplitude of alternate second-harmonic swings introduced a fundamental component into the output which was of the proper phase to create the normal double-sideband envelope.

Typical examples of these waveforms, at the common plate terminal of the two modulator tubes, are shown in the photographs of Figures 13-16. Photographs mounted side-by-side are for the same conditions, with oscilloscope sweep normal and expanded. The lower waveform of each photograph is the corresponding double-sideband envelope output from the band-pass filter (12.5-15.7 kc.), which, with a carrier frequency of 14 kilocycles, allowed only the upper and lower sidebands to pass, attenuating all other signal components.

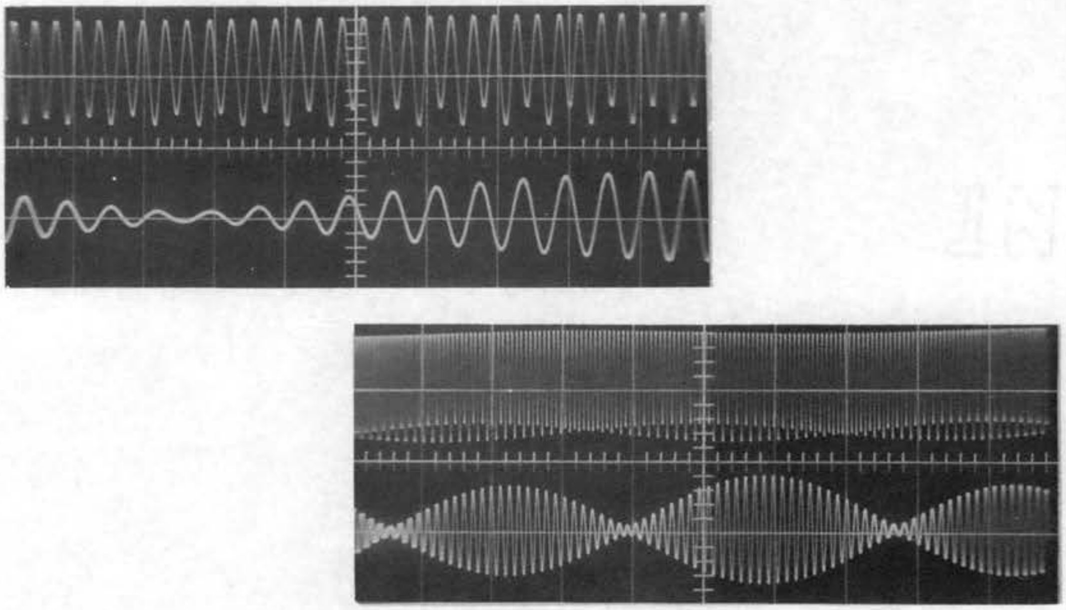

Figure 13. Low Modulation. $e_m = 1$ volt, $e_c = 1$ volt.
10 v/cm. top trace, 0.05 v/cm. lower.

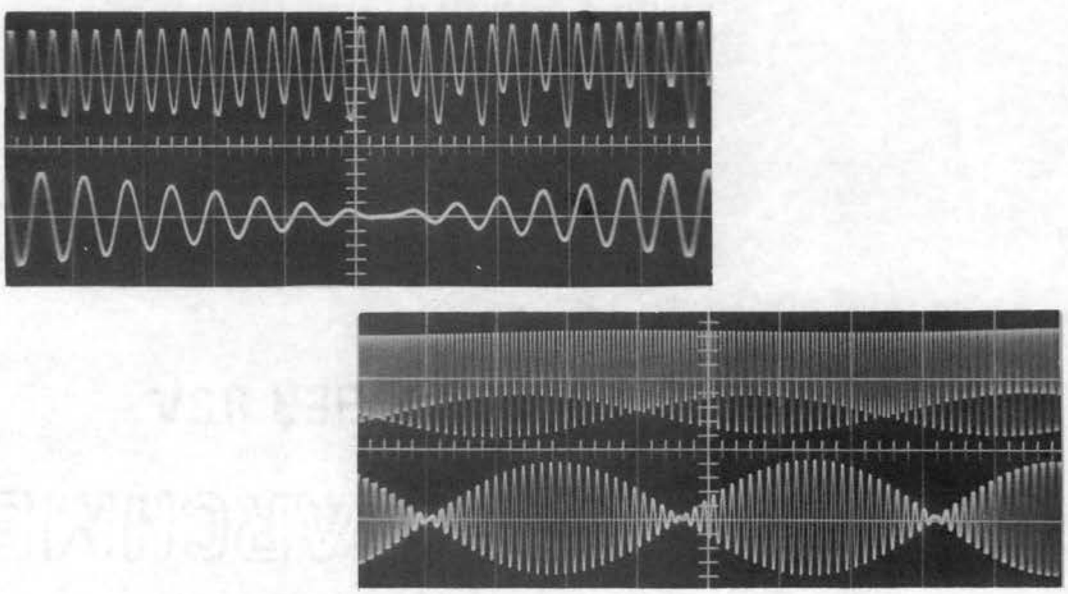

Figure 14. Medium Modulation. $e_m = 5.6$ volts, $e_c = 2.5$ volts.
50 v/cm top trace, 0.5 v/cm lower.

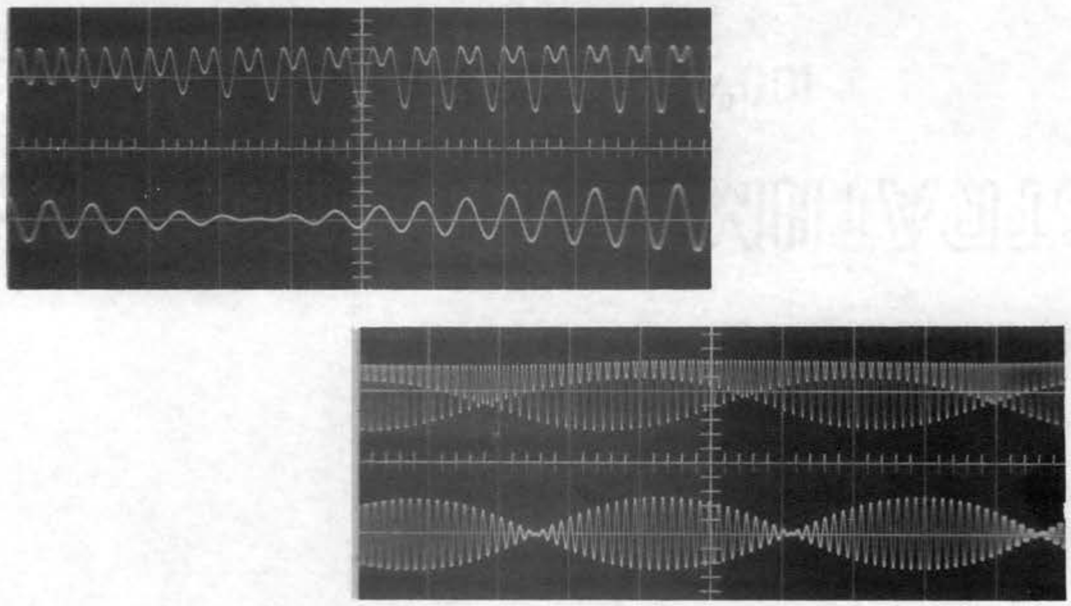

Figure 15. High Modulation. $e_m = 15$ volts, $e_c = 2.5$ volts. 100 v/cm. top trace, 2 v/cm lower.

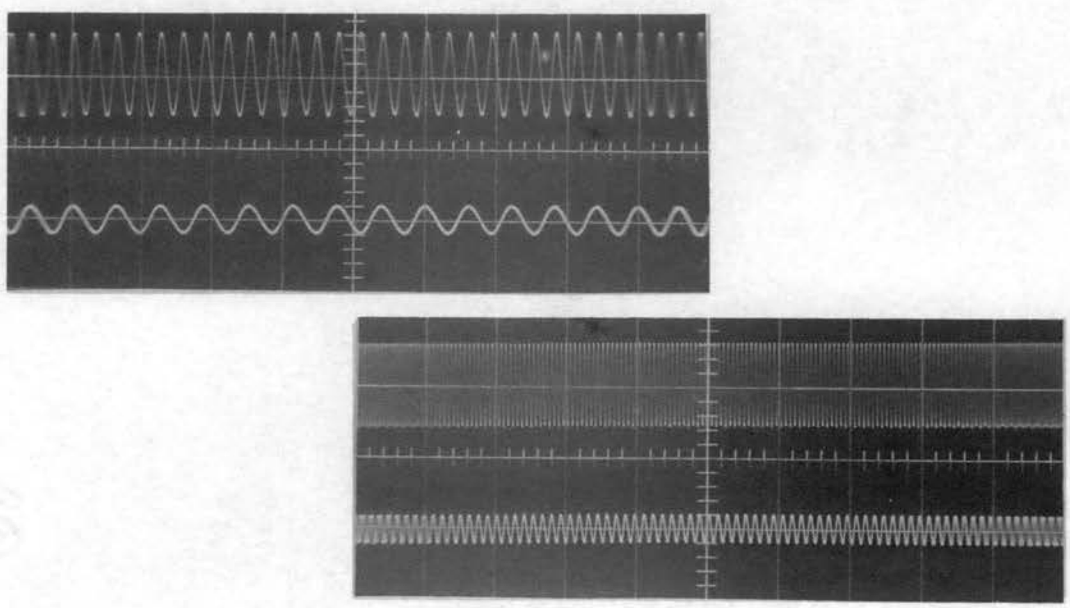

Figure 16. No Modulation. $e_m = 0$ volt, $e_c = 2.5$ volts. 50 v/cm. 0.05 v/cm. Modulator unbalanced.

No quantitative tests were run, but a number of observations on the "mixing" or frequency-changing ability of this circuit were made. For the input-signal frequency range of 1 to 40 kilocycles, any combination of frequencies which gave a sum or difference frequency of 14 kilocycles provided a sizable output from the filter, which was approximately the same for any frequency combination in this range. The amplitude of these sum or difference frequencies was approximately the same as the sideband signals produced when the circuit was used for normal modulation.

As a frequency doubler, the circuit performed throughout the range of 100 cycles to 25 kilocycles, although the waveform of the second harmonic component tended to distort at the lower frequencies.

CHAPTER VI

SUMMARY AND CONCLUSIONS

The results of this investigation, in general, indicate that the circuit investigated is not suitable for use as a balanced modulator in any system where extreme carrier suppression is of importance. However, general performance of the circuit as a modulator is good. In an application where extreme carrier suppression is not required, it would seem to be a valuable circuit. On this basis, use of this circuit in frequency converter or product detector applications would be recommended. Although it does not provide adequate carrier suppression for critical applications, the degree of carrier cancellation is remarkable for a circuit which does not require push-pull inputs. On this basis alone, it is useful in many applications.

The performance of the circuit as a frequency-doubler is amazing, in that very high second harmonic output of good waveform is obtained--without the use of tuned circuits. This feature would seem to have intriguing possibilities at audio frequencies, where frequency-doubling with good waveform is a problem.

General results of the nonlinear analysis were disappointing. However, several assumptions were made in the devlopment of this analysis which undoubtedly contributed the majority of this error. Probably the most serious of these was the assumption that the only changes in r_p resulted from input voltage effects--ignoring the second-order effects of

r_p itself. It is the feeling of this investigator that the general approach utilized in this analysis offers promise in the analysis of nonlinear electrical devices and circuits--particularly those involving vacuum tubes. The selected perturbation of appropriate parameters in a linear analysis offers the advantage of relatively direct identification of terms with their physical significance in the circuit. Thus, simplifications, as required for algebraic solution, can be based on the physical significance of the items involved. The analysis in this report is a first attempt at this approach--and gave results which left much to be desired. However, with some experience to serve as a guide--particularly in the matter of appropriate and realistic simplifications and assumptions--it is felt that the method offers some promise. On the basis of no more than an "intuitive guess", this method might prove to be a successful approach for use with "iteration" techniques on a digital computer, by which simplifications could be held to a minimum.

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APPENDIX A

DETAILED LINEAR ANALYSIS

The linear equivalent circuit to be analyzed is duplicated here for convenience.

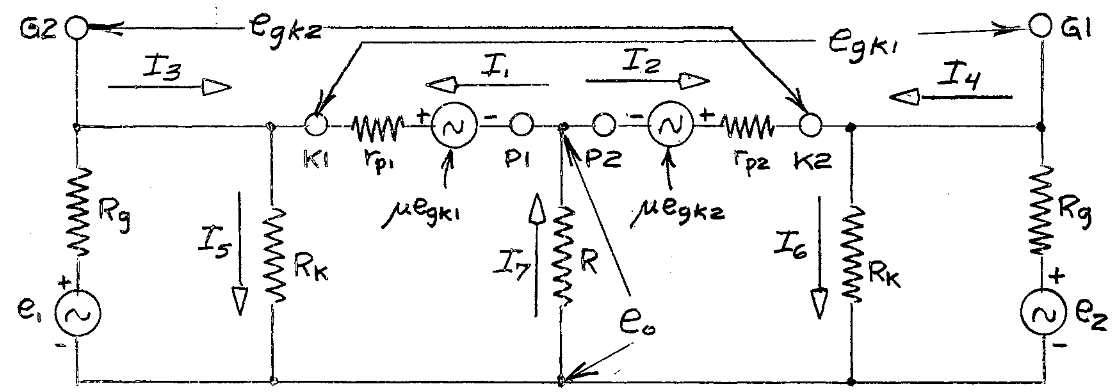

Figure A-1. Modulator Equivalent Circuit

For this circuit the following current-node and voltage-loop equations can be written:

$$I_1 + I_3 = I_5, \tag{1}$$

$$I_2 + I_4 = I_6, \tag{2}$$

$$I_1 + I_2 = I_7, \tag{3}$$

$$-e_1 + I_3R_g + I_5R_k = 0, \tag{4}$$

$$-e_2 + I_4R_g + I_6R_k = 0, \tag{5}$$

$$I_5R_k + I_7R + I_1r_{p1} - \mu_1e_{gk1} = 0, \tag{6}$$

$$I_6R_k + I_7R + I_2r_{p2} - \mu_2e_{gk2} = 0, \tag{7}$$

$$e_{gk2} - I_4R_g + e_2 - e_1 + I_3R_g = 0, \tag{8}$$

$$e_{gk1} - I_3R_g + e_1 - e_2 + I_4R_g = 0. \tag{9}$$

The first seven equations of this group form a linearly independent set in the seven current variables. The last two equations will be used to eliminate the "dependent" voltage generators, e_{gk1} and e_{gk2}, from the independent set.

Solving [8] and [9] for the e_{gk}'s in terms of the other variables yields

$$e_{gk2} = e_1 - e_2 + I_4R_g - I_3R_g, \qquad [10]$$

$$e_{gk1} = e_2 - e_1 + I_3R_g - I_4R_g. \qquad [11]$$

Substituting [10] and [11] into [6] and [7] gives

$$I_5R_k + I_7R + I_1r_{p1} - \mu_1e_1 + \mu_1e_2 - \mu_1I_4R_g + \mu_1I_3R_g = 0, \qquad [12]$$

$$I_6R_k + I_7R + I_2r_{p2} - \mu_2e_2 + \mu_2e_1 - \mu_2I_3R_g + \mu_2I_4R_g = 0. \qquad [13]$$

These two equations along with equations [1] through [5] comprise the independent set.

Substituting equations [1], [2], and [3] into [4], [5], [12], and [13] to eliminate the variables I_5, I_6, and I_7, gives

$$-e_1 + I_3R_g + I_1R_k + I_3R_k = 0, \qquad [14]$$

$$-e_2 + I_4R_g + I_2R_k + I_4R_k = 0, \qquad [15]$$

$$I_1R_k + I_3R_k + I_1R + I_2R + I_1r_{p1} - \mu_1e_1 + \mu_1e_2$$
$$-\mu_1I_4R_g + \mu_1I_3R_g = 0, \qquad [16]$$

$$I_2R_k + I_4R_k + I_1R + I_2R + I_2r_{p2} - \mu_2e_2 + \mu_2e_1$$
$$-\mu_2I_3R_g + \mu_2I_4R_g = 0. \qquad [17]$$

Solving [14] and [15] for I_3 and I_4, respectively, gives

$$I_3(R_g + R_k) = e_1 - I_1R_k$$

$$I_4(R_g + R_k) = e_2 - I_2R_k,$$

$$\text{or,}\qquad I_3 = \frac{e_1 - I_1 R_k}{R_g + R_k} \qquad [18]$$

$$I_4 = \frac{e_2 - I_2 R_k}{R_g + R_k}\,. \qquad [19]$$

Rearranging [16] and [17] by collecting terms in the I's yields

$$I_1\,(R_k + R + r_{p1}) + I_2 R + I_3 (R_k - \mu_1 R_g) + I_4 (\mu_1 R_g) = \mu_1 e_2 - \mu_1 e_1 \qquad [20]$$

$$I_1 R + I_2 (R_k + R + r_{p2}) + I_3 (\mu_2 R_g) + I_4 (R_k - \mu_2 R_g) = \mu_2 e_1 - \mu_2 e_2 \qquad [21]$$

Substituting [18] and [19] into [20] and [21] gives

$$I_1 (R_k + R + r_{p1}) + I_2 R + (e_1 - I_1 R_k)\left(\frac{R_k - \mu_1 R_g}{R_g + R_k}\right)$$
$$+ (e_2 - I_2 R_k)\ \frac{\mu_1 R_g}{R_g + R_k} = \mu_1 e_2 - \mu_1 e_1 \qquad [22]$$

$$I_1 R + I_2 (R_k + R + r_{p2}) + (e_1 - I_1 R_k)\left(\frac{\mu_2 R_g}{R_g + R_k}\right)$$
$$+ (e_2 - I_2 R_k)\left(\frac{R_k - \mu_2 R_g}{R_g + R_k}\right) = \mu_2 e_1 - \mu_2 e_2\,. \qquad [23]$$

Collecting terms and simplifying, [22] and [23] become

$$I_1 \left[r_{p1} + R + (\mu_1 + 1)\frac{R_g R_k}{R_g + R_k}\right] + I_2 \left[R - \mu_1 \frac{R_g R_k}{R_g + R_k}\right] = \mu_1 \frac{R_k}{R_g + R_k} - (\mu_1 + 1)\frac{R_k}{R_g + R_k}\, e_1 \qquad [24]$$

$$I_1 \left[R - \mu_2\, \frac{R_g R_k}{R_g + R_k}\right] + I_2 \left[r_{p2} + R + (\mu_2 + 1)\frac{R_g R_k}{R_g + R_k}\right] = \mu_2\, \frac{R_k}{R_g + R_k}\, e_1 - (\mu_2 + 1)\frac{R_k}{R_g + R_k}\, e_2\,. \qquad [25]$$

Let:

$$R_1 = R + (\mu_1 + 1)\, \frac{R_g R_k}{R_g + R_k}\,, \qquad [26a]$$

$$R_2 = R - \mu_1\, \frac{R_g R_k}{R_g + R_k}\,, \qquad [26b]$$

$$R_3 = R + (\mu_2 + 1)\, \frac{R_g R_k}{R_g + R_k}\,, \qquad [26c]$$

$$R_4 = R - \mu_2\, \frac{R_g R_k}{R_g + R_k}\,, \qquad [26d]$$

$$b = \frac{R_k}{R_g + R_k}\,. \qquad [26e]$$

Substituting [26] into [24] and [25] gives

$$I_1[r_{p1} + R_1] + I_2R_2 = \mu_1 be_2 - (\mu_1 + 1)be_1 \qquad [27]$$

$$I_1R_4 + I_2[r_{p2} + R_2] = \mu_2 be_1 - (\mu_2 + 1)be_2. \qquad [28]$$

Solving [27] and [28] for I_2, equating the resulting expressions, and rearranging gives

$$I_1[R_2R_4 - (r_{p1} + R_1)(r_{p2} + R_3)] =$$

$$[\mu_2 bR_2 + (\mu_1 + 1)\, b\, (r_{p2} + R_3)]\, e_1$$

$$-[\mu_1 b\, (r_{p2} + R_3) + (\mu_2 + 1)\, bR_2]\, e_2,$$

or,

$$I_1 = \frac{\mu_2 bR_2 + (\mu_1 + 1)\, b\, (r_{p2} + R_3)}{R_2R_4 - (r_{p1} + R_1)(r_{p2} + R_3)}\, e_1$$

$$- \frac{(\mu_2 + 1)\, bR_2 + \mu_1\, b\, (r_{p2} + R_3)}{R_2R_4 - (r_{p1} + R_1)(r_{p2} + R_3)}\, e_2. \qquad [29]$$

Similarly, it is found that

$$I_2 = \frac{\mu_1\, b\, R_4 + (\mu_2 + 1)\, b\, (r_{p1} + R_1)}{R_2R_4 - (r_{p1} + R_1)(r_{p2} + R_3)}\, e_2$$

$$- \frac{(\mu_1 + 1)\, b\, R_4 + \mu_2\, b\, (r_{p1} + R_1)}{R_2R_4 - (r_{p1} + R_1)(r_{p2} + R_3)}\, e_1. \qquad [30]$$

If it is assumed that

$$\mu_1 = \mu_2 = \mu,$$

it is found that

[31]

$$R_1 = R_3,$$

$$R_2 = R_4.$$

Inspection of the equivalent circuit gives

$$e_o = - I_7R = - (I_1 + I_2)\ R. \qquad [32]$$

Substituting [29] and [30] into [32], with the assumptions set forth in [31], yields

$$e_o = \frac{bR}{R_2^{\,2} - (r_{p1} + R_1)\ (r_{p2} + R_1)} \Big[[(\mu + 1)\ r_{p2} - \mu r_{p1} + R_1 - R_2]\ e_1 + [(\mu + 1)\ r_{p1} - \mu r_{p2} + R_1 - R_2]\ e_2 \Big]. \qquad [33]$$

From [26], the value of $(R_1 - R_2)$ may be simplified to

$$R_1 - R_2 = (\mu + 1)\ \frac{R_gR_k}{R_g+R_k} + \mu\ \frac{R_gR_k}{R_g+R_k},$$

or,

$$R_1 - R_2 = (2\mu+1)\ \frac{R_gR_k}{R_g+R_k} = (2\mu + 1)\ b\ R_g. \qquad [34]$$

Substituting [34] into [33] yields the final form of the expression for the output voltage e_o, namely

$$e_o = \frac{bR}{R_2^{\,2} - (r_{p1} + R_1)\ (r_{p2} + R_1)} \Big[[(\mu+1) r_{p2} - \mu r_{p1} + (2\mu+1)\ bR_g]\ e_1 + [(\mu + 1) r_{p1} - \mu r_{p2} + (2\mu + 1)\ bR_g]\ e_2 \Big]. \qquad [35]$$

APPENDIX B

DETAILED NONLINEAR ANALYSIS

The nonlinear analysis of this circuit will be based on a "perturbed" extension of the linear analysis of Appendix A. As explained in the body of the report, the "perturbation" will be introduced by representing the plate resistances of the tubes in the form

$$r_p = r - kI, \qquad [1]$$

in which r is the plate resistance of the tube at the quiescent point, and k is the slope of the $r_p - i_b$ curve at that point.

From [1] the two r_p's of the circuit may be represented in the form

$$r_{p1} = r - kI_1, \qquad [2]$$

and

$$r_{p2} = r - kI_2, \qquad [3]$$

since the tubes are assumed to be identical.

Since the tubes are assumed to be identical, and the μ is very little changed by changes in plate current, equations [29] and [30] of Appendix A may be written in the form

$$I_1 = \frac{\mu bR_2 + (\mu + 1)\ b\ (r_{p2} + R_1)}{R_2^2 - (r_{p1} + R_1)\ (r_{p2} + R_1)}\ e_1$$

$$- \frac{(\mu + 1)\ bR_2 + \mu b\ (r_{p2} + R_1)}{R_2^2 - (r_{p1} + R_1)\ (r_{p2} + R_1)}\ e_2, \qquad [4]$$

and,

$$I_1 = \frac{\mu b R_2 + (\mu + 1)\, b\, (r_{p1} + R_1)}{R_2^2 - (r_{p1} + R_1)\,(r_{p2} + R_1)}\, e_2$$

$$- \frac{(\mu + 1)\, b R_2 + \mu b\, (r_{p1} + R_1)}{R_2^2 - (r_{p1} + R_1)\,(r_{p2} + R_1)}\, e_1, \qquad [5]$$

since $\mu_1 = \mu_2 = \mu$, $R_3 = R_1$, and $R_4 + R_2$.

As can be seen, since [4] and [5] involve r_{p1} and r_{p2}, direct substitution of these relationships into [2] and [3] will introduce higher-order terms in the r_p's. Further analysis along these lines is difficult, since it requires simultaneous solution of two quadratic equations in two unknowns.

Thus, a <u>most important</u> assumption will be made concerning the expressions at [4] and [5]. It will be assumed that the only variations in I_1 and I_2 expressed by [4] and [5] will result from the signal voltages e_1 and e_2, and that the r_p's in [4] and [5] will remain constant, at value <u>r</u>. This assumption is expressed mathematically by writing [4] and [5] in the form

$$I_1 = k_1 e_1 - k_2 e_2, \qquad [6]$$

and

$$I_2 = k_1 e_2 - k_2 e_1, \qquad [7]$$

where

$$k_1 = b\, \frac{\mu R_2 + (\mu + 1)\,(r + R_1)}{R_2^2 - (r + R_1)^2}, \qquad [8]$$

and

$$k_2 = b\, \frac{(\mu + 1)\, R_2 + \mu (r + R_1)}{R_2^2 - (r + R_1)^2}. \qquad [9]$$

Substituting [6] and [7] into [2] and [3], respectively, yields

$$r_{p1} = r - k\ (k_1e_1 - k_2e_2) = r - k_3e_1 + k_4e_2, \qquad [10]$$

and

$$r_{p2} = r - k\ (k_1e_2 - k_2e_1) = r - k_3e_2 + k_4e_1, \qquad [11]$$

where

$$k_3 = kk_{17} \text{ and } k_4 = kk_2.$$

The expressions for the "perturbed" values of r_{p1} and r_{p2} at [10] and [11] can now be introduced into the output equation developed by linear circuit analysis.

The output of the circuit is

$$e_o = \frac{bR}{R_e^2}\Big[[(\mu + 1)\ r_{p2} - \mu r_{p1} + (2\mu + 1)\ b\ R_g]e_1$$
$$+ [(\mu + 1)\ r_{p1} - \mu r_{p2} + (2\mu + 1)\ b\ R_g]e_2\Big], \qquad [12]$$

where

$$R_e^2 = R_2^2 - (r_{p1} + R_1)\ (r_{p2} + R_1).$$

As a first step in this analysis, it is convenient to evaluate R_e^2 with the "nonlinear" r_p's at [10] and [11] introduced.

Carrying out the multiplications indicated in the equation for R_e^2 yields

$$R_e^2 = R_2^2 - r_{p1}r_{p2} - R_1r_{p1} - R_1r_{p2} - R_1^2,$$

or

$$R_e^2 = (R_2^2 - R_1^2) - R_1(r_{p1} + r_{p2}) - r_{p1}r_{p2}. \qquad [13]$$

Substituting [10] and [11] into [13] gives

$$R_e^2 = (R_2^2 - R_1^2) - R_1(r - k_3e_1 + k_4e_2 + r - k_3e_2 + k_4e_1)$$
$$- (r - k_3e_1 + k_4e_2)\ (r - k_3e_2 + k_4e_1),$$

or

$$R_e^2 = (R_2^2 - R_1^2 - 2\ R_1\ r - r^2) + (k_3 - k_4)\ (R_1 + r)e_1$$
$$+(k_3 - k_4)\ (R_1 + r)e_2 + (k_3^2 + k_4^2)e_1e_2 - k_3k_4e_1^2 - k_3k_4e_2^2.$$
$$[14]$$

For algebraic convenience, [14] may be written in the form

$$R_e^2 = k_5 + k_6e_1 + k_6e_2 + k_7e_1e_2 + k_8e_1^2 + k_8e_2^2, \qquad [15]$$

where

$$k_5 = (R_2^2 - R_1^2 - 2\,R_1\,r - r^2),$$

$$k_6 = (k_3 - k_4)\,(R_1 + r),$$

$$k_7 = (k_3^2 + k_4^2),$$

and

$$k_8 = -\,k_3k_4.$$

Introducing [10] and [11] into [12] yields

$$e_o = \frac{bR}{R_e^2}\Big[[(\mu+1)(r - k_3e_2 + k_4e_1) - \mu(r - k_3e_1 + k_4e_2) + (2\mu+1)\,b\,R_g]e_1 + [(\mu+1)\,(r - k_3e_1 + k_4e_2) - \mu(r - k_3e_2 + k_4e_1) + (2\mu+1)\,b\,R_g]e_2\Big]. \qquad [16]$$

After expanding, simplifying, and rearranging terms in the brackets, [16] becomes

$$e_o = \frac{bR}{R_e^2}\Big[[(\mu k_3 + \mu k_4 + k_4)e_1 - (\mu k_3 + \mu k_4 + k_3)e_2]e_1 + [r + (2\mu+1)\,b\,R_g]e_1 + [(\mu k_3 + \mu k_4 + k_4)e_2 - (\mu k_3 + \mu k_4 + k_3)e_1]e_2 + [r + (2\mu+1)\,b\,R_g]e_2\Big]. \qquad [17]$$

Rearranging the terms of [17] as coefficients of various functions of e_1 and e_2 gives

$$e_o = \frac{bR}{R_e^2}\Big[[r + (2\mu+1)\,b\,R_g]e_1 + [r + (2\mu+1)\,b\,R_g]e_2 - 2[\mu k_3 + \mu k_4 + k_3]e_1e_2 + [\mu k_3 + \mu k_4 + k_4]e_1^2 + [\mu k_3 + \mu k_4 + k_4]e_2^2\Big]. \qquad [18]$$

[18] may be written in the form

$$e_o = \frac{bR}{R_e^2}\left[k_9e_1 + k_9e_2 + k_{10}e_1e_2 + k_{11}e_1^2 + k_{11}e_2^2\right], \qquad [19]$$

where

$$k_9 = [r + (2\mu + 1)\, b\, R_g],$$

$$k_{10} = -2[\mu k_3 + \mu k_4 + k_3],$$

and

$$k_{11} = [\mu k_3 + \mu k_4 + k_4].$$

Substituting the value of R_e^2 at [15] into [19] yields

$$e_o = bR\left[\frac{k_9e_1 + k_9e_2 + k_{10}e_1e_2 + k_{11}e_1^2 + k_{11}e_2^2}{k_5 + k_6e_1 + k_6e_2 + k_7e_1e_2 + k_8e_1^2 + k_8e_2^2}\right],$$

or

$$e_o = \frac{bR}{k_5}\left[\frac{k_9e_1 + k_9e_2 + k_{10}e_1e_2 + k_{11}e_1^2 + k_{11}e_2^2}{1 + k_{12}e_1 + k_{12}e_2 + k_{13}e_1e_2 + k_{14}e_1^2 + k_{14}e_2^2}\right], \qquad [20]$$

where

$$k_{12} = \frac{k_6}{k_5},\ k_{13} = \frac{k_7}{k_5},\ \text{and}\ k_{14} = \frac{k_8}{k_5}.$$

By a process of simple division, [20] may be obtained in the form

$$\begin{aligned} e_o = \frac{bR}{k_5}\Big[& k_9e_1 + k_9e_2 + (k_{10} - 2\,k_9k_{12})e_1e_2 \\ & + (k_{11} - k_9k_{12})e_1^2 + (k_{11} - k_9k_{12})e_2^2 \\ & - [k_9(k_{13} + k_{14}) + k_{12}\,(k_{10} + k_{11} - 3\,k_9k_{12})]e_1^2e_2 \\ & - [k_9(k_{13} + k_{14}) + k_{12}\,(k_{10} + k_{11} - 3\,k_9k_{12})]e_1e_2^2 \\ & - [k_9k_{14} + k_{12}\,(k_{11} - k_9k_{12})]e_1^3 \\ & - [k_9k_{14} + k_{12}\,(k_{11} - k_9k_{12})]e_2^3 \\ & + \cdots\ \text{higher-order terms}\Big], \end{aligned}$$

or

$$e_o = \frac{bR}{k_5}\Big[k_9e_1 + k_9e_2 + (k_{10} - 2\ k_9k_{12})e_1e_2$$

$$+ (k_{11} - k_9k_{12})e_1{}^2 + (k_{11} - k_9k_{12})e_2{}^2$$

$$+ k_{15}\ e_1{}^2e_2 + k_{15}\ e_1e_2{}^2$$

$$+ k_{16}\ e_1{}^3 + k_{16}\ e_2{}^3$$

$$+ \cdots \text{ higher-order terms}\Big], \qquad [21]$$

where

$$k_{15} = -[k_9(k_{13} + k_{14}) + k_{12}(k_{10} - 2\ k_9k_{12}) + k_{12}\ (k_{11} - k_9k_{12})],$$

and

$$k_{16} = -[k_9k_{14} + k_{12}\ (k_{11} - k_9k_{12})].$$

[21] is the final result of the nonlinear analysis, into which signal voltages of any desired form can be introduced.

APPENDIX C

EXPERIMENTAL MEASUREMENTS

Data collected by experimental measurements was in three major categories. They are:

1. Spectrum-analysis. Measurement of the signal voltage (rms) present on the various frequencies (harmonic and sideband) at and above the carrier frequency, for a variety of applied carrier and modulation voltage levels. Measurements were made with the Hewlett-Packard Type 302A Wave Analyzer, using the Hewlett-Packard Type 200AB Audio Oscillator for carrier and modulation voltage generators.
2. Electrode DC Potentials. Measurement of the d.c. voltages present on the various electrodes of the two tubes in the modulator circuit for a variety of applied carrier and modulation voltage levels. Measurements were made with the Hewlett-Packard Type 412A Vacuum-Tube DC Voltmeter, using the Hewlett-Packard Type 200AB Audio Oscillator for carrier and modulation voltage generators.
3. Carrier Signal Leak-through. Measurement of the signal voltage (rms) present on the carrier frequency, for a variety of applied carrier and modulation voltage levels, under various conditions of initial balance for minimum carrier leak-through. Measurements made with the H-P

Audio Oscillators, as in Data Group 1.

Visual observation of the plate waveform (output) and the filtered (12.5-15.7 kc.) double-sideband output of the modulator were made at the time of the voltage measurements. A number of these waveforms were photographed, typical examples having been presented in the body of this report. As a part of the measurement and observation devices, a cathode-follower stage was constructed to drive the band-pass filter (600 ohm input and output impedance) and provide isolation of these measurements from the unfiltered output (at the plate connection) of the modulator.

Data, as collected in sections 1, 2, and 3, is presented in Tables 1, 2, and 3 of this Appendix.

TABLE I

SPECTRUM ANALYSIS

All readings are in r.m.s. volts.

1. Modulation voltage input = 0 volts.

Carrier Input	Output ω_c	$\omega_c+\omega_m$	$\omega_c+2\omega_m$	$\omega_c+3\omega_m$	$2\omega_c$	$2\omega_c+\omega_m$	$3\omega_c$	$3\omega_c+\omega_m$
0.5	0.67				1.60		0.02	
1.0	0.82				5.60		0.11	
2.0	0.27				17.0		0.23	
3.0	0.23				25.5		0.65	
4.0	0.54				31.0		0.83	
5.0	0.90				35.0		1.00	
6.0	1.20				38.0		1.10	
7.0	1.50				39.5		1.20	
8.0	1.80				40.5		1.30	
9.0	1.88				42.0		1.4+	
10.0	2.10				42.5		1.55	

2. Modulation voltage input = 0.5 volts.

0.5	0.62	0.12			1.5			
1.0	0.76	0.24			5.7	0.005		
2.0	0.21	0.44			17.0	0.020		
3.0	0.22	0.51			26.0	0.025		
4.0	0.43	0.54			100	0.025		
5.0	0.79	0.54			115	0.025		
6.0	1.10	-			120			
7.0	1.40	-			-			
8.0	1.65	-			-			
9.0	1.85	-			-			
10.0	2.05	-			-			

TABLE I (Continued)

3. Modulation voltage input = 1.0 volts.

Carrier Input	Output							
	ω_c	$\omega_c+\omega_m$	$\omega_c+2\omega_m$	$\omega_c+3\omega_m$	$2\omega_c$	$2\omega_c+\omega_m$	$3\omega_c$	$3\omega_c+\omega_m$
1.0	0.82	0.46	-	0.0025	6.1	0.007	0.11	0.008
1.5	0.45	0.70	-	0.0035	12.0	0.015	0.15	0.021
2.0	0.25	0.85	-	0.004	17.0	0.035	0.24	0.065
2.5	0.21	0.93	-	0.0047	22.0	0.045	0.49	0.095
3.0	0.22	1.00	-	0.005	25.5	0.050	0.67	0.110
3.5	0.32	1.05	-	0.005	28.5	0.050	0.76	0.110
4.0	0.48	1.05	-	0.005	32.0	0.053	0.85	0.120
4.5	0.66	1.05	-	0.005	34.0	0.053	0.92	0.120

4. Modulation voltage input = 1.5 volts.

1.0	0.77	0.68	-	0.003	6.0	0.012	0.115	0.012
1.5	0.42	1.05	-	0.005	12.0	0.025	0.150	0.035
2.0	0.21	1.25	-	0.006	17.0	0.050	0.240	0.100
2.5	0.23	1.40	-	0.007	22.0	0.065	0.500	0.140
3.0	0.22	1.45	-	0.007	26.0	0.073	0.650	0.160
3.5	0.26	1.50	-	0.007	29.0	0.080	0.770	0.165
4.0	0.42	1.52	-	0.007	31.0	0.080	0.830	0.170

5. Modulation voltage input = 2.0 volts.

1.0	0.82	0.90	-	0.004	6.1	0.015	0.110	0.016
1.5	0.44	1.40	-	0.007	12.0	0.035	0.160	0.045
2.0	0.23	1.65	-	0.008	17.0	0.070	0.240	0.130
2.5	0.21	1.85	-	0.009	22.0	0.090	0.480	0.190
3.0	0.22	1.95	-	0.009	25.5	0.100	0.650	0.210
3.5	0.31	2.00	-	0.009	28.5	0.105	0.770	0.220
4.0	0.47	2.00	-	0.010	31.0	0.110	0.860	0.250

TABLE I (Continued)

6. Modulation voltage input = 2.5 volts.

Carrier Input	Output				
	ω_c	$\omega_c+\omega_m$	$\omega_c+2\omega_m$	$2\omega_c$	$3\omega_c$
0.5		0.52			
1.0		1.10			
1.5		1.70			
2.0		2.20			
2.5		2.30			
3.0		2.40			
3.5		2.48			
4.0		2.53			
4.5		2.55			

7. Modulation voltage input = 3.5 volts.

0.5		0.73			
1.0		1.62			
1.5		2.40			
2.0		2.90			
2.5		3.20			
3.0		3.40			
3.5		3.50			
4.0		3.60			

8. Modulation voltage input = 5.0 volts.

0.5	0.57	1.05	0.011	1.40	0.009
1.0	0.75	2.25	0.020	5.30	0.040
1.5	0.41	3.50	0.013	11.20	0.105
2.0	0.19	4.20	0.008	17.0	0.330
2.5	0.16	4.60	0.008	21.5	0.470
3.0	0.20	4.90	0.010	25.0	0.530
3.5	0.31	5.00	0.011	28.0	0.540
4.0	0.45	5.10	0.013	31.0	0.550

TABLE I (Continued)

9. Modulation input 6.0 volts.

Carrier Input	Output $\omega_c+\omega_m$
0.5	1.15
1.0	2.70
1.5	4.10
2.0	5.00
2.5	5.40
3.0	5.80
3.5	6.00
4.0	6.10

10. Modulation input 7.0 volts.

Carrier Input	Output $\omega_c+\omega_m$
0.5	1.35
1.0	3.10
1.5	4.80
2.0	5.80
2.5	6.40
3.0	6.80
3.5	7.00
4.0	7.10

11. Modulation input 8.0 volts.

0.5	1.60
1.0	3.50
1.5	4.40
2.0	6.60
2.5	7.40
3.0	7.70
3.5	8.00
4.0	8.10

12. Modulation input 9.0 volts.

0.5	1.80
1.0	4.00
1.5	6.20
2.0	7.50
2.5	8.30
3.0	8.70
3.5	9.00
4.0	9.20

13. Modulation input 10.0 volts.

0.5	1.90
1.0	4.40
1.5	6.80
2.0	8.20
2.5	9.00
3.0	9.60
3.5	10.00
4.0	10.10

14. Modulation input 11.0 volts

0.5	2.20
1.0	4.90
1.5	7.40
2.0	9.00
2.5	10.00
3.0	10.70
3.5	11.00
4.0	11.40

15. Modulation input 12.0 volts.

0.5	2.3
1.0	5.2
1.5	8.0
2.0	10.0
2.5	11.0
3.0	11.7
3.5	12.0
4.0	12.3

16. Modulation input 13.0 volts.

0.5	2.55
1.0	5.6
1.5	8.5
2.0	10.9
2.5	12.0
3.0	12.7
3.5	13.0
4.0	13.3

TABLE I (Continued)

17. Modulation input
14.0 volts.

Carrier Input	Output $\omega_c+\omega_m$
0.5	2.75
1.0	6.1
1.5	9.4
2.0	11.5
2.5	12.9
3.0	13.5
3.5	14.1
4.0	14.3

18. Modulation input
15.0 volts.

Carrier Input	Output $\omega_c+\omega_m$
0.5	2.95
1.0	6.7
1.5	10.1
2.0	12.3
2.5	13.8
3.0	14.5
3.5	15.0
4.0	15.3

18. Modulation input
16.0 volts.

0.5	3.2
1.0	7.1
1.5	10.7
2.0	13.0
2.5	14.6
3.0	15.4
3.5	16.0
4.0	16.4

19. Modulation input
17.0 volts.

0.5	3.5
1.0	7.3
1.5	11.4
2.0	14.0
2.5	15.6
3.0	16.4
3.5	17.0
4.0	17.3

20. Modulation input
18.0 volts.

0.5	3.6
1.0	7.6
1.5	12.0
2.0	14.7
2.5	16.2
3.0	17.1
3.5	17.9
4.0	18.2

21. Modulation input
19.0 volts.

0.5	3.8
1.0	8.4
1.5	12.7
2.0	15.5
2.5	17.0
3.0	18.1
3.5	18.8
4.0	19.0

22. Modulation input = 20.0 volts.

Carrier Input	Output $\omega_c+\omega_m$	Output $\omega_c+2\omega_m$
0.5	4.2	0.19
1.0	8.6	0.28
1.5	13.2	0.22
2.0	16.0	0.17
2.5	18.0	0.19
3.0	19.1	0.23
3.5	19.8	0.25
4.0	20.1	0.26

TABLE II

ELECTRODE DC POTENTIALS

1. Carrier voltage, 1.0 volts, injected at pin 8.
No balance resistor.

Tube pin #	Modulation input voltage		
	0 volts	10 volts	20 volts
1-6	201	202	201
2	23.3	23.3	23.3
3	26.1	26.2	26.2
7	21.2	21.2	21.2
8	24.1	24.2	24.3

2. Carrier voltage, 2.0 volts, injected at pin 8.
No balance resistor.

1-6	198	198	202
2	23.2	23.0	21.3
3	26.8	26.6	25.2
7	22.9	22.7	20.2
8	26.6	26.2	24.3

3. Carrier voltage, 1.0 volts, injected at pin 8.
Balance resistor (330 ohm) in pin 1.

1-6	202	202	201
2	23.3	23.3	23.4
3	26.0	26.2	26.3
7	21.0	21.0	21.0
8	24.0	24.0	24.2

4. Carrier voltage, 2.0 volts, injected at pin 8.
Balance resistor in pin 1.

1-6	198	198	202
2	23.3	23.0	21.2
3	26.7	26.6	25.0
7	22.8	22.6	20.2
8	26.4	26.2	24.3

TABLE II (Continued)

5. Carrier voltage, 1.0 volts, injected at pin 3.
No balance resistor.

Tube pin #	Modulation input voltage: 0 volts	10 volts	20 volts
1-6	198	198	198
2	23.4	23.4	23.3
3	26.2	26.3	26.4
7	23.5	23.5	23.4
8	26.0	26.1	26.2

6. Carrier voltage, 2.0 volts, injected at pin 3.
No balance resistor.

1-6	197	197	197
2	23.3	23.2	21.8
3	26.9	26.9	25.7
7	23.7	23.6	22.0
8	27.0	26.9	25.7

7. Carrier voltage, 1.0 volts, injected at pin 3.
Balance resistor in pin 1.

1-6	199	198	199
2	23.4	23.4	23.4
3	26.3	26.3	26.4
7	23.2	23.2	23.2
8	25.7	25.8	26.0

8. Carrier voltage, 2.0 volts, injected at pin 3.
Balance resistor in pin 1.

1-6	197	198	199
2	23.5	23.5	21.8
3	27.0	27.0	25.7
7	23.8	23.7	22.1
8	27.0	27.0	25.8

TABLE III

CARRIER SIGNAL LEAK-THROUGH

1. Carrier voltage, 1.0 volts, injected at pin 3. Balanced for carrier signal minimum at modulation voltage of 0 volts.

Modulation Voltage	Carrier leak-through amplitude volts	db.
0	0.038	
2	0.040	
4	0.047	
6	0.064	
8	0.084	
10	0.115	
12	0.150	
14	0.195	
16	0.230	
18	0.280	
20	0.330	

2. Balanced for carrier minimum with modulation 10 volts

0	0.105	
2	0.100	
4	0.090	
6	0.075	
8	0.060	
10	0.053	
12	0.067	
14	0.095	
16	0.135	
18	0.180	
20	0.230	

3. Balanced for carrier minimum with modulation 20 volts

0	0.280	-9.0
2	0.280	-9.0
4	0.270	-9.5
6	0.250	-10.0
8	0.230	-11.0
10	0.200	-12.0
12	0.160	-13.0
14	0.140	-15.0
16	0.110	-17.0
18	0.095	-18.0
20	0.105	-17.0

TABLE III (Continued)

4. Carrier voltage, 2.0 volts, injected at pin 3.
Balanced for carrier minimum with modulation 0 v.

Modulation Voltage	Carrier leak-through amplitude volts	db
0	0.043	-25.0
2	0.042	-25.0
4	0.055	-23.0
6	0.085	-19.0
8	0.135	-15.0
10	0.200	-12.0
12	0.265	-9.5
14	0.330	-7.0
16	0.370	-6.0
18	0.380	-6.0
20	0.340	-7.0

5. Balanced for carrier minimum with modulation 10 v.

0	0.180	-12.5
2	0.170	-13.0
4	0.150	-14.0
6	0.120	-16.0
8	0.080	-20.0
10	0.080	-20.0
12	0.140	-15.0
14	0.205	-11.5
16	0.260	-9.5
18	0.280	-9.0
20	0.270	-9.0

6. Balanced for carrier minimum with modulation 20 v.

0	0.540	-3.0
2	0.540	-3.0
4	0.500	-4.0
6	0.460	-4.5
8	0.400	-6.0
10	0.300	-8.0
12	0.205	-12.0
14	0.140	-15.0
16	0.140	-15.0
18	0.170	-13.0
20	0.190	-12.0

VITA

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